Exponent Laws for Multiplying and Dividing Powers with the Same Base

Question	Long Expansion	Short Expansion
1. $(-5)^2 \times (-5)^4$	$\underbrace{(-5)(-5)}\times\underbrace{(-5)(-5)(-5)(-5)}=(-5)^6$	$(-5)^{2+4}=(-5)^6$
2. $7^5 \div 7^3$	$\frac{\cancel{7}\times\cancel{7}\times\cancel{7}\times 7\times 7}{\cancel{7}\times\cancel{7}\times\cancel{7}} = 7^2$	$7^{5-3}=7^2$
3. $\frac{(y^5)(y^2)}{y^4}$	$\frac{\cancel{(y)}\cancel{(y)}\cancel{(y)}\cancel{(y)}(y)\underbrace{(y)(y)}}{\underbrace{\cancel{(y)}\cancel{(y)}\cancel{(y)}\cancel{(y)}}} = y^3$	$y^{(5+2)-4}=y^{7-4}=y^3$
4. $\frac{\left[(x^4y)(xy^3)\right]}{x^3y^2}$	$\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot y\cdot x\cdot y\cdot\cancel{y}\cdot\cancel{y}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{y}\cdot\cancel{y}} = x^2y^2$	$\frac{x^{4+1}y^{1+3}}{x^3y^2}=\frac{x^5y^4}{x^3y^2}=x^{5-3}y^{4-2}=x^2y^2$

$a^m \times a^n =$ a^{m+n}. To multiply powers with the **same base**, Keep the base and add the exponents.

$a^m \div a^n =$ a^{m-n}. To divide powers with the **same base**, Keep the base and Subtract the exponents.

Ex. 1. Simplify to a single power.

a) $(2^{15})(2^{25})$
$= 2^{15+25}$
$= 2^{40}$

b) $\frac{x^{18}}{x^{12}}$
$= x^{18-12}$
$= x^6$

c) $3^{-18}\times 3^{-13}$
$= 3^{-18+(-13)}$
$= 3^{-18-13}$
$= 3^{-31}$

d) $y^{-10}\div y^{-6}$
$= y^{-10-(-6)}$
$= y^{-10+6}$
$= y^{-4}$

e) $6^{-\frac{1}{3}}\times 6^{0.5}$
$= 6^{-\frac{1}{3}}\times 6^{\frac{1}{2}}$
$= 6^{-\frac{1}{3}+\frac{1}{2}}$
$= 6^{-\frac{2}{6}+\frac{3}{6}}$
$= 6^{\frac{1}{6}}$

f) $\left(\frac{8}{9}\right)^{15}\left(\frac{8}{9}\right)^{10}\div\left(\frac{8}{9}\right)^{5}$
$= \left(\frac{8}{9}\right)^{15+10-5}$
$= \left(\frac{8}{9}\right)^{20}$

g) $(4^5)(x^8)(4^1)(x^2)(x^1)$
$= 4^{5+1}x^{8+2+1}$
$= 4^6x^{11}$

h) $\frac{\left[(-3)^7(x^8)(x^{16})\right]}{\left[(-3)^1(x^{10})(x^1)(-3)^2\right]}$
$= \frac{(-3)^7x^{8+16}}{(-3)^{1+2}x^{10+1}}$
$= \frac{(-3)^7x^{24}}{(-3)^3x^{11}}$ or $= \frac{(-3)^7}{(-3)^3}\cdot\frac{x^{24}}{x^{11}}$
$= (-3)^{7-3}x^{24-11}$
$= (-3)^4x^{13}$

Ex. 2. Simplify to a single power, and then evaluate.

a) $\frac{4^5}{4^2}$

$= 4^{5-2}$

$= 4^3$

$= 64$

b) $(10^3)(10^2)$

$= 10^{3+2}$

$= 10^5$

$= 100\ 000$

c) $(-3)^{-1.2}(-3)^{5.2}$

$= (-3)^{-1.2+5.2}$

$= (-3)^{4.0}$

$= (-3)^4$

$= +81$

d) $\frac{6^{\sqrt{100}}}{6^{\sqrt{49}}}$

$= \frac{6^{10}}{6^7}$

$= 6^{10-7}$

$= 6^3$

$= 216$

$\begin{array}{r} 36 \\ \times 6 \\ \hline \end{array}$

e) $\left(-\frac{2}{3}\right)^7 \div \left(-\frac{2}{3}\right)^5\left(-\frac{2}{3}\right)^1$

$= \left(-\frac{2}{3}\right)^{7-5+1}$

$= \left(-\frac{2}{3}\right)^3$

$= \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$

$= -\frac{8}{27}$

f) $\frac{(5^3)(3)^1(5^4)(3^8)}{(3^2)(5^6)(3^5)}$

$= \frac{5^{3+4} \cdot 3^{1+8}}{5^6 \cdot 3^{2+5}}$

$= \frac{5^7}{5^6} \cdot \frac{3^9}{3^7}$

$= 5^{7-6} \cdot 3^{9-7}$

$= 5^1 \cdot 3^2$

$= 5 \cdot 9$

$= 45$

Ex. 3. Simplify using the exponent laws, and then evaluate for $x = 5$ and $y = -2$.

a) $\frac{(y^7)(y^3)}{(y^1)(y^4)}$

$= \frac{y^{10}}{y^5}$

$= y^5$

if $y = -2$

$= (-2)^5$

$= -32$

b) $\frac{-5x^5y^6}{25x^4y^2}$

$= -\frac{5}{25} \cdot \frac{x^5}{x^4} \cdot \frac{y^6}{y^2}$

$= -\frac{1}{5} \cdot x^1 \cdot y^4$

if $x = 5, y = -2$

$= -\frac{1}{5} \cdot (5) \cdot (-2)^4$

$= -\frac{1}{\cancel{5}_1} \cdot \frac{\cancel{5}^1}{1} \cdot \frac{16}{1}$

$= -16$

c) $\frac{(4^5)(x^4)(y^2)(x^6)}{(y)(x^8)(4^4)}$

$= \frac{4^5 \cdot x^{4+6} \cdot y^2}{4^4 \cdot x^8 \cdot y^1}$

$= \frac{4^5}{4^4} \cdot \frac{x^{10}}{x^8} \cdot \frac{y^2}{y^1}$

$= 4^{5-4} \cdot x^{10-8} \cdot y^{2-1}$

$= 4^1 \cdot x^2 \cdot y^1$

$= 4^1 \cdot (5)^2 (-2)^1$

$= 4(25)(-2)$

$= -200$

Exponent Law for Power of a Power

Warm-up: **i)** $a^m \times a^n = a^{m+n}$ **ii)** $\frac{a^m}{a^n} = a^{m-n}$

1. Simplify to a single power, and then evaluate.

a) $(4^8)(4^{-5})$
$= 4^{8+(-5)}$
$= 4^{8-5}$
$= 4^3$
$= 64$

b) $2^{1.4} \times 2^{2.8} \times 2^{1.8}$
$= 2^{1.4+2.8+1.8}$
$= 2^{6.0}$
$= 2^6$
$= 64$

c) $\frac{(1.1)^{0.8}}{(1.1)^{-1.2}}$
$= (1.1)^{0.8-(-1.2)}$
$= (1.1)^{0.8+1.2}$
$= (1.1)^2$
$= 1.21$

$11 \times 11 = 121$

d) $5^{\frac{5}{6}} \div 5^{\frac{1}{3}} \times 5^{\frac{1}{2}}$
$= 5^{\frac{5}{6}-\frac{1}{3}+\frac{1}{2}}$
$= 5^{\frac{5}{6}-\frac{2}{6}+\frac{3}{6}}$
$= 5^1$
$= 5$

e) $\left(-1\frac{1}{2}\right)^{-3} \div \left(-1\frac{1}{2}\right)^{5} \div \left(-1\frac{1}{2}\right)^{-11}$
$= \left(-\frac{3}{2}\right)^{-3-5-(-11)}$
$= \left(-\frac{3}{2}\right)^{-3-5+11}$
$= \left(-\frac{3}{2}\right)^3$
$= -\frac{27}{8}$
$= -3\frac{3}{8}$

2. Simplify using the exponent laws and express the final answer using scientific notation.

a) $(5\times10^7)(6\times10^{13})$
$= 5\times6\times10^7\times10^{13}$
$= 30\times10^{20}$
$= 3\times10^1\times10^{20}$
$= 3.0\times10^{21}$

b) $\frac{6\times10^{15}}{1.5\times10^4}$
$= \frac{6_{\times10}}{1.5_{\times10}}\times\frac{10^{15}}{10^4}$
$= \frac{60}{15}\times10^{15-4}$
$= 4.0\times10^{11}$

3. Simplify the following expressions using the exponent laws and evaluate for $x=-1$ and $y=4$.

a) $\frac{36x^6y^7}{-9xy^4}$
$= -\frac{36}{9}\cdot\frac{x^6}{x^1}\cdot\frac{y^7}{y^4}$
$= -4x^5y^3$
$= -4(-1)^5(4)^3$
$= -4(-1)(64)$
$= +4\times1\times64$
$= 256$

b) $\frac{(-2x^5)(6x^6y^4)}{(4y)(x^3y^2)}$
$= \frac{-2\cdot6\cdot x^5\cdot x^6\cdot y^4}{4\cdot x^3\cdot y^1\cdot y^2}$
$= -\frac{12}{4}\cdot\frac{x^{11}}{x^3}\cdot\frac{y^4}{y^3}$
$= -3x^8y^1$
$= -3(-1)^8(4)^1$
$= -3\times1\times4$
$= -12$

Power	Long Expansion	Short Expansion
$(2^2)^5$	$2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 = 2^{2+2+2+2+2}$ $= 2^{10}$	$2^{2\cdot 5} = 2^{10}$
$(y^5)^4$	$(y^5)(y^5)(y^5)(y^5) = y^{5+5+5+5}$ $= y^{20}$	$y^{5\cdot 4} = y^{20}$

$(a^m)^n =$ $a^{m\cdot n}$ To simplify a ***power of a power*** Keep the base, multiply the exponents.

Ex. 1. Simplify to a power with a single exponent.

a) $(2^4)^8$

$= 2^{4\cdot 8}$

$= 2^{32}$

b) $\left(x^{\frac{1}{2}}\right)^{10}$

$= x^{\frac{1}{\cancel{2}_1}\cdot \cancel{10}^{5}}$

$= x^5$

c) $\left[(9^2)^3\right]^4$

$= 9^{2\cdot 3\cdot 4}$

$= 9^{24}$

d) $(5^{-3})^3(5^{-2})^{-7}$

$= 5^{(-3)(3)} \cdot 5^{(-2)(-7)}$

$= 5^{-9}\cdot 5^{14}$

$= 5^{-9+14}$

$= 5^5$

e) $\left(\dfrac{a^8}{a^4}\right)^6$

$= (a^{8-4})^6$

$= (a^4)^6$

$= a^{4\cdot 6}$

$= a^{24}$

f) $\dfrac{(y^2 y^4)^8}{(y^4 y)^7}$

$\boxed{= \dfrac{(y^{2+4})^8}{(y^{4+1})^7}}$

$= \dfrac{(y^6)^8}{(y^5)^7}$

$\boxed{= \dfrac{y^{6\cdot 8}}{y^{5\cdot 7}}}$

$= \dfrac{y^{48}}{y^{35}}$

$\boxed{= y^{48-35}}$

$= y^{13}$

Ex. 2. Simplify to a single power, and then evaluate.

a) $\dfrac{(4^3)^5(4^3)^2}{(4^9)^2}$

$= \dfrac{4^{15}\cdot 4^6}{4^{18}}$

$= \dfrac{4^{21}}{4^{18}}$

$= 4^3 = 64$

b) $\dfrac{(6^{-4}\times 6)^{-5}}{(6^5\times 6^2)^2}$

$= \dfrac{(6^{-3})^{-5}}{(6^7)^2}$

$= \dfrac{6^{15}}{6^{14}}$

$= 6^1$

$= 6$

c) $\dfrac{(5^2)^5(3^2)}{(3^{-1})^2(5^3)^3}$

$= \dfrac{3^2}{3^{-2}}\cdot\dfrac{5^{10}}{5^9}$

$= 3^{2-(-2)}\cdot 5^{10-9}$

$= 3^{2+2}\cdot 5^1$

$= 3^4\cdot 5^1$

$= 81\cdot 5$

$= 405$

Ex. 3. Simplify using the exponent laws, and then evaluate for $y = -2$.

$\dfrac{(y^4)^2}{(y^3)^2} - \dfrac{(y^6)^3}{(y^5)^3}$

$= \dfrac{y^8}{y^6} - \dfrac{y^{18}}{y^{15}}$

NO exponent law for add/subtract powers with same base

$= y^2 - y^3$

$= (-2)^2 - (-2)^3$

$= 4 - (-8)$

$= 4 + 8$

$= 12$

Ex. 4. Express 64 as a power of base:

i) 8

64

$= 8^2$

ii) 4

64

$= 4^3$

iii) 2

64

$= 2^6$

Exponent Laws for Power of a Product and Quotient

multiply divide

Warm-up: **i)** $a^m \times a^n = a^{m+n}$ **ii)** $\dfrac{a^m}{a^n} = a^{m-n}$ **iii)** $(a^m)^n = a^{m\cdot n}$

1. Simplify to a single power, and then evaluate.

a) $\left(\dfrac{3^{\frac{1}{3}} \times 3^{\frac{1}{2}}}{3^{-\frac{5}{6}}}\right)^3$

$= \left(3^{\frac{1}{3}+\frac{1}{2}-\left(-\frac{5}{6}\right)}\right)^3$

$= \left(3^{\frac{2}{6}+\frac{3}{6}+\frac{5}{6}}\right)^3$

$= \left(3^{\frac{10}{6}}\right)^3$

$= \left(3^{\frac{5}{3}}\right)^3$

$= 3^{\frac{5}{\cancel{3}_1}\times\frac{\cancel{3}^1}{1}}$

$= 3^5$

$= 243$

b) $\left[\left(4^2\right)^3\right]^4 - \left(4^{-3}\right)^{-8}$

$= 4^{2\cdot3\cdot4} - 4^{(-3)(-8)}$

$= 4^{24} - 4^{24}$

$= 0$

2. Express as a power of the base indicated.

a) 16^5 with base 2

16^5

$= (16)^5$

$= (2^4)^5$

$= 2^{4\cdot5}$

$= 2^{20}$

b) 27^3 with base 3

27^3

$= (27)^3$

$= (3^3)^3$

$= 3^{3\cdot3}$

$= 3^9$

Power	Long Expansion	Short Expansion
$(xy)^6$	$xy \cdot xy \cdot xy \cdot xy \cdot xy \cdot xy = x^6y^6$	$(xy)^6 = x^6y^6$
$\left(\dfrac{x}{y}\right)^4$	$\dfrac{x}{y}\cdot\dfrac{x}{y}\cdot\dfrac{x}{y}\cdot\dfrac{x}{y} = \dfrac{x^4}{y^4}$	$\left(\dfrac{x}{y}\right)^4 = \dfrac{x^4}{y^4}$
$\left(\dfrac{x^2y^3}{z^4}\right)^3$	$\dfrac{x^2y^3}{z^4}\cdot\dfrac{x^2y^3}{z^4}\cdot\dfrac{x^2y^3}{z^4} = \dfrac{x^6y^9}{z^{12}}$	$\left(\dfrac{x^2y^3}{z^4}\right)^3 = \dfrac{(x^2)^3(y^3)^3}{(z^4)^3} = \dfrac{x^6y^9}{z^{12}}$

$(a^mb)^n = a^{m\cdot n}b^n$ To simplify a ***power of a product***, share the exponent with **each** factor of the base.

$\left(\dfrac{ab^m}{c^md}\right)^n = \dfrac{a^nb^{m\cdot n}}{c^{m\cdot n}d^n}$ To simplify a ***power of a quotient***, share the exponent with **each** factor in the numerator and denominator.

Note: $\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4} = \dfrac{16}{81}$

$\left(-\dfrac{2}{3}\right)^4 = \dfrac{(-2)^4}{(3)^4} = \dfrac{16}{81}$

Ex. 1. Simplify the following algebraic expressions using the exponent laws.

a) $(-2a^3)^5$

$=(-2)^5(a^3)^5$

$=(-2)^5 a^{15}$

or

$=-32a^{15}$

b) $(2^2m^4n)^3$

$=(2^2)^3(m^4)^3(n^1)^3$

$=2^6m^{12}n^3$

or $=64m^{12}n^3$

c) $(4a^2\times 3b^4)^2$

$=(12a^2b^4)^2$

$=(12)^2(a^2)^2(b^4)^2$

$=12^2a^4b^8$

or

$=144a^4b^8$

d) $\dfrac{(3^6\times 4^5)^3}{(4^7\times 3^8)^2}$

$=\dfrac{(3^6)^3(4^5)^3}{(4^7)^2(3^8)^2}$

$=\dfrac{3^{18}}{3^{16}}\cdot\dfrac{4^{15}}{4^{14}}$

$=3^2\cdot 4^1$

$=9\cdot 4$

$=36$

e) $(5x^5)^3(-3x^7)^2$

$=(5^1)^3(x^5)^3(-3)^2(x^7)^2$

$=5^3(-3)^2\cdot x^{15}\cdot x^{14}$

$=125\cdot 9\cdot x^{29}$

$=1125x^{29}$

$\begin{array}{r}125\\ \times 9\\ \hline\end{array}$

f) $\dfrac{-24x^{-4}y^5}{8x^{-5}y^3}$

$=\dfrac{-24}{8}\cdot\dfrac{x^{-4}}{x^{-5}}\cdot\dfrac{y^5}{y^3}$

$=-3x^{-4-(-5)}y^2$

$=-3x^{-4+5}y^2$

$=-3x^1y^2$

g) $\left(\dfrac{10x^3y^2}{-5xy^{-1}}\right)^4$

$=\left(\dfrac{10}{-5}\cdot\dfrac{x^3}{x^1}\cdot\dfrac{y^2}{y^{-1}}\right)^4$

$=\left(-2x^2y^{2-(-1)}\right)^4$

$=(-2x^2y^3)^4$

$=(-2)^4(x^2)^4(y^3)^4$

$=16x^8y^{12}$

h) $\left(\dfrac{-1a^3b}{3c^5}\right)^3$

$=\dfrac{(-1)^3(a^3)^3(b)^3}{(3)^3(c^5)^3}$

$=\dfrac{-1a^9b^3}{27c^{15}}$

$=\dfrac{-a^9b^3}{27c^{15}}$

Ex. 2. Simplify using the exponent laws, and then evaluate for $a=3$ and $b=-2$.

a) $\dfrac{\left(\frac{2}{3}a^2b^4\right)^4}{\left(\frac{2}{3}a^2b^5\right)^3}$

$=\dfrac{\left(\frac{2}{3}\right)^4(a^2)^4(b^4)^4}{\left(\frac{2}{3}\right)^3(a^2)^3(b^5)^3}$

$=\dfrac{\left(\frac{2}{3}\right)^4a^8b^{16}}{\left(\frac{2}{3}\right)^3a^6b^{15}}$

$=\left(\frac{2}{3}\right)^1a^2b^1$

$=\left(\frac{2}{3}\right)(3)^2(-2)$

$=\frac{2}{3}\cdot\frac{9}{1}\cdot\frac{(-2)}{1}$

$=-\frac{2}{\cancel{3}_1}\cdot\frac{\cancel{9}^3}{1}\cdot\frac{2}{1}$

$=-12$

b) $(-3a^{-11}b^{-3})(-1a^2b)^6$

$=(-3)(a^{-11})(b^{-3})(-1)^6(a^2)^6(b)^6$

$=(-3)(-1)^6\cdot a^{-11}\cdot a^{12}\,b^{-3}\cdot b^6$

$=-3\,a^1b^3$

$=-3(3)^1(-2)^3$

$=-3(3)(-8)$

$=+72$

Zero and Negative Exponents

Warm-up: **i)** $a^m \times a^n = a^{m+n}$ **ii)** $\frac{a^m}{a^n} = a^{m-n}$ **iii)** $(a^m)^n = a^{m\cdot n}$ **iv)** $\left(\frac{ab^m}{c^m d}\right)^n = \frac{a^n b^{m\cdot n}}{c^{m\cdot n} d^n}$

1. Simplify using the exponent laws, and then evaluate.

a) $\left(\frac{3^8 \times 2^4}{6^2}\right)^{\frac{1}{2}}$

$= \frac{(3^8)^{\frac{1}{2}} \times (2^4)^{\frac{1}{2}}}{(6^2)^{\frac{1}{2}}}$

$= \frac{3^4 \times 2^2}{6}$

$= \frac{\overset{27}{\cancel{81}} \times \overset{2}{\cancel{4}}}{\cancel{6}\ \cancel{3}\ 1}$

$= \frac{54}{1}$

$= 54$

b) $\frac{(5^3)^{-3}(5^{-4})}{(5^4)^{-4}}$

$= \frac{(5^{-9})(5^{-4})}{5^{-16}}$

$= \frac{5^{-9+(-4)}}{5^{-16}}$

$= \frac{5^{-13}}{5^{-16}}$

be careful

$= 5^{-13-(-16)}$

$= 5^{-13+16}$

$= 5^3$

$= 125$

2. Simplify to a single power of base 2, and then evaluate.

$\frac{32^5}{4^2 \times 16^4}$

$= \frac{(2^5)^5}{(2^2)^2 \times (2^4)^4}$

$= \frac{2^{25}}{2^4 \times 2^{16}}$

$= \frac{2^{25}}{2^{20}}$

$= 2^5$

$= 32$

$\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$

Power	**Value**	**Power with a positive exponent**
3^3	27	3^3
3^2	9	3^2
$3^1 = 3$	3	3^1
3^0	1	
3^{-1}	$\frac{1}{3}$	$\frac{1}{3^1}$ or $\left(\frac{1}{3}\right)^1$
3^{-2}	$\frac{1}{9}$	$\frac{1}{3^2}$ or $\left(\frac{1}{3}\right)^2$
3^{-3}	$\frac{1}{27}$	$\frac{1}{3^3}$ or $\left(\frac{1}{3}\right)^3$

More Exponent Laws

i) $a^0 = 1$

ii) $a^{-n} = \frac{1}{a^n}$ or $\left(\frac{1}{a}\right)^n$

iii) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

To evaluate a ***power*** *with a* ***negative exponent****, take the* reciprocal *of the base in brackets and raise it to the* positive *exponent.*

Zero and Negative Exponent Laws

i) $a^0 = 1$ $\qquad (-a)^0 = 1$ $\qquad -a^0 = -1$

ii) $a^{-n} = \frac{1}{a^n}$ $\qquad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\qquad \frac{1}{a^{-n}} = a^n$

Ex. 1. Evaluate each of the following using the exponent laws.

a) 2^{-4}

$= \left(\frac{1}{2}\right)^4$ or $\frac{1}{2^4}$

$= \frac{1}{16}$

b) $\left(\frac{1}{3}\right)^{-4}$

$= \frac{3^4}{1}$

$= 81$

c) $3^0 - \frac{1}{4^{-2}}$

$= 1 - 4^2$

$= 1 - 16$

$= -15$

d) $-3^0 + 5^{-1}$

$= -1 + \frac{1}{5^1}$

$= -\frac{1}{1} + \frac{1}{5}$

$= -\frac{5}{5} + \frac{1}{5}$

$= -\frac{4}{5}$

e) $(-3)^{-3} - 9^{-1}$

$= \frac{1}{(-3)^3} - \frac{1}{9^1}$

$= -\frac{1}{27} - \frac{1}{9}$

$= -\frac{1}{27} - \frac{3}{27}$

$= -\frac{4}{27}$

f) $\left(1 - \frac{2}{5}\right)^{-3}$

$= \left[\frac{5}{5} - \frac{2}{5}\right]^{-3}$

$= \left(\frac{3}{5}\right)^{-3}$

$= \left(\frac{5}{3}\right)^3$

$= \frac{125}{27}$

g) $\dfrac{2^{-3} - 2^{-1}}{4^{-1} + 4^{-2}}$

$= \dfrac{\frac{1}{2^3} - \frac{1}{2^1}}{\frac{1}{4^1} + \frac{1}{4^2}}$

$= \dfrac{\left[\frac{1}{8} - \frac{1}{2}\right]}{\left[\frac{1}{4} + \frac{1}{16}\right]}$

$= \dfrac{\frac{1}{8} - \frac{4}{8}}{\frac{4}{16} + \frac{1}{16}}$

$= \frac{-3}{8} \div \frac{5}{16}$

$= -\frac{3}{\cancel{8}_1} \times \frac{\cancel{16}^2}{5}$

$= -\frac{6}{5}$

Ex. 2. Simplify to a single power, and then evaluate.

a) $2^{-4} \div 2^{1} \div 2^{-11}$

$= 2^{-4-1-(-11)}$

$= 2^{-4-1+11}$

$= 2^6$

$= 64$

b) $\left(-\frac{2}{5}\right)^8 \times \left(-\frac{2}{5}\right)^0 \div \left(-\frac{2}{5}\right)^{11}$

$= \left(-\frac{2}{5}\right)^{8+0-11}$

$= \left(-\frac{2}{5}\right)^{-3}$

$= \left(-\frac{5}{2}\right)^3$

$= -\frac{125}{8}$

Negative Exponents and Scientific Notation

Scientific Notation: is a way of writing an extremely large or extremely small number as a decimal between 1 and 10, multiplied by a power of 10.

speed of light
300 000 000 *m/s*
$= 3.0 \times 10^8$

mass of a proton
0.000 000 000 000 000 000 000 001 76 *grams*
$= 1.76 \times 10^{-24}$

Ex. 1. Express in standard form.

a) 1.6×10^5
$= 160\ 000$

b) 3.85×10^{-2}
$= 0.0385$

c) 2.1×10^{-8}
$= 0.000\ 000\ 021$

Ex. 2. Express in scientific notation.

a) 5 300 000
$= 5.3 \times 10^6$

b) 0.000 000 802
$= 8.02 \times 10^{-7}$

c) 54.1×10^{-5}
$= \underbrace{5.41 \times 10^1}\times 10^{-5}$
$= 5.41 \times 10^{-4}$

d) 0.282×10^9
$= \underbrace{2.82 \times 10^{-1}} \times 10^9$
$= 2.82 \times 10^8$

Ex. 3. Express each number in scientific notation, simplify using the exponent laws, and then evaluate. Leave your final answer in scientific notation.

a) $(40\ 000\ 000\ 000)(0.000\ 032)$
$= (4 \times 10^{10})(3.2 \times 10^{-5})$
$= 4 \times 3.2 \times 10^{10} \times 10^{-5}$
$= 12.8 \times 10^5$
$= 1.28 \times 10^1 \times 10^5$
$= 1.28 \times 10^6$

b) $(0.000\ 032) \div (40\ 000\ 000\ 000)$
$= \dfrac{3.2 \times 10^{-5}}{4 \times 10^{10}}$
$= \dfrac{3.2}{4} \times \dfrac{10^{-5}}{10^{10}}$
$= 0.8 \times 10^{-15}$
$= 8.0 \times 10^{-1} \times 10^{-15}$
$= 8.0 \times 10^{-16}$

$$\begin{array}{r} 0.8 \\ 4\overline{)\,3.2} \\ -3\,2 \\ \hline 0 \end{array}$$

Ex. 4. According to the U.S. National Debt Clock, the outstanding Public Debt is about 16 trillion dollars. If the estimated population of the United States is 320 million, determine each citizen's share of of the debt.

$\dfrac{16 \text{ trillion}}{320 \text{ million}}$
$= \dfrac{16\ 000\ 000\ 000\ 000}{320\ 000\ 000}$
$= \dfrac{1.6 \times 10^{13}}{3.2 \times 10^8}$
$= \dfrac{1.6 \times 10}{3.2 \times 10} \times \dfrac{10^{13}}{10^8}$
$= \dfrac{16}{32} \times \dfrac{10^{13}}{10^8}$
$= \dfrac{1}{2} \times \dfrac{10^{13}}{10^8}$
$= 0.5 \times 10^5$
$= 5.0 \times 10^{-1} \times 10^5$
$= 5.0 \times 10^4$
$= 50\ 000$

$\therefore$ each citizen's share of the debt is \$50 000.

Negative Exponents and Algebraic Expressions

Simplifying Algebraic Expressions:

Ex. 1. Simplify using the exponent laws, and then express your final answer with only positive exponents, if applicable.

a) $x^2 \cdot x^{-5} \div x^1$

$= x^{2+(-5)-1}$

$= x^{-4}$

$= \frac{1}{x^4}$

b) $(-4n^1)(-3n^3)(-1n^{-12})$

$= (-4)(-3)(-1)(n^1)(n^3)(n^{-12})$

$= -12n^{1+3+(-12)}$

$= -12n^{-8}$

$= -\frac{12}{1} \cdot \frac{1}{n^8}$

$= -\frac{12}{n^8}$

c) $(8a^{-3}) \div (12a^{-4})$

$= \frac{8a^{-3}}{12a^{-4}}$

$= \frac{8}{12} \cdot \frac{a^{-3}}{a^{-4}}$

$= \frac{2}{3} \cdot a^{-3-(-4)}$

$= \frac{2}{3}a^{-3+4}$

$= \frac{2}{3}a$

or $\frac{2a}{3}$

d) $(3a^2b^5)(2ab^{-8})$

$= (3)(2)a^{2+1}b^{5+(-8)}$

$= 6a^3b^{-3}$

$= \frac{6}{1} \cdot \frac{a^3}{1} \cdot \frac{1}{b^3}$

$= \frac{6a^3}{b^3}$

e) $-36x^3y^4 \div (-9x^{-1}y^4)$

$= \frac{-36x^3y^4}{-9x^{-1}y^4}$

$= \frac{-36}{-9} \cdot \frac{x^3}{x^{-1}} \cdot \frac{y^4}{y^4}$

$= 4 \cdot x^{3-(-1)} \cdot y^{4-4}$

$= 4x^4y^0$

$= 4x^4(1)$

$= 4x^4$

f) $(-2m^{-5}n^{-3})^3$

$= (-2)^3(m^{-5})^3(n^{-3})^3$

$= -8 \cdot m^{-15}n^{-9}$

$= -\frac{8}{1} \cdot \frac{1}{m^{15}} \cdot \frac{1}{n^9}$

$= \frac{-8}{m^{15}n^9}$

g) $[2x^2(-3x^4)]^{-2}$

$= (-6x^6)^{-2}$

$= (-6)^{-2}(x^6)^{-2}$

$= \frac{1}{(-6)^2} \cdot x^{-12}$

$= \frac{1}{36} \cdot \frac{1}{x^{12}}$

$= \frac{1}{36x^{12}}$

h) $\frac{5c^{-1}d^7}{(-3c^4d^{-2})^2}$

$= \frac{5c^{-1}d^7}{(-3)^2(c^4)^2(d^{-2})^2}$

$= \frac{5c^{-1}d^7}{9c^8d^{-4}}$

$= \frac{5}{9} \cdot \frac{c^{-1}}{c^8} \cdot \frac{d^7}{d^{-4}}$

$= \frac{5}{9}c^{-1-8}d^{7-(-4)}$

$= \frac{5}{9} \cdot c^{-9}d^{11}$

$= \frac{5}{9} \cdot \frac{1}{c^9} \cdot \frac{d^{11}}{1}$

$= \frac{5d^{11}}{9c^9}$

i) $\left(\frac{4m^4n^{-9}}{mn^{-2}}\right)^{-3}$

$= (4m^3n^{-7})^{-3}$

$= (4)^{-3}(m^3)^{-3}(n^{-7})^{-3}$

$= \frac{1}{4^3} \cdot m^{-9} \cdot n^{21}$

$= \frac{1}{64} \cdot \frac{1}{m^9} \cdot \frac{n^{21}}{1}$

$= \frac{n^{21}}{64m^9}$