Communicate With Algebra

Terms and Degrees of Terms:

A ***term*** is an expression formed by the **product** of **numbers** and/or **variables** . In the **term** $-3x^2y$,

-3 is the coefficient, and x^2y is the variable part.

The ***degree of a term*** is the sum of the exponents on the variable(s)

Ex. 1. Identify the coefficient and the variable part of each term and state the degree of the term.

Term	Coefficient	Variable Part	Degree
$1x^2$	1	x^2	2
$3y^4$	3	y^4	4
$0.7u^1$	0.7	u	1
$-2a^2b^1$	-2	a^2b	3
$\frac{2}{3}x^1y^1$	$\frac{2}{3}$	xy	2
$-5x^0$ $= -5(1) = -5$	-5	none	0

constant term

Polynomials and Degrees of Polynomials:

One term is called a monomial eg. $2m^2n^3$

Two terms connected by + or – signs is called a binomial eg. $2x^2 - 5x$

Three terms connected by + or – signs is called a trinomial eg. $3x^2 - 2x + 7$

Many terms connected by + or – signs is called a polynomial eg. $4a - 5b + 6c + 8d$

The ***degree of a polynomial*** is the degree of the highest degree term.

Ex. 2. Classify each polynomial by the number of terms it has and state the degree of the polynomial.

Polynomial	Type of Polynomial	Degree of Polynomial
$3x^2y^2 + 2x^2y$	binomial	4
$2m^2n^3$	monomial	5
$3y^3 + 0.2y - 8$	trinomial	3
$a - 2b + c - 3$	(4-term) polynomial	1
-0.5	monomial	0

Using an Algebraic Model to Solve a Problem:

Ex. 3. A small pizza costs $7.00 plus $1.50 per topping.

a) Define the variable and write an expression that describes the total cost of a small pizza in terms of the variable.

Let x represent the number of toppings.

Total Cost: $1.50x + 7.00$

or $1.5x + 7$

b) Determine the cost of a small pizza with 5 toppings.

$1.5x+7$

Sub in $x=5$

$1.5(5)+7$

$=7.5+7$

$=14.5$

∴ the cost with 5 toppings is $14.50

Ex. 4. Cheryl works part-time as a ski instructor. She earns $125 for the season, plus $20 for each children's lesson and $30 for each adult lesson that she gives.

a) Define the variables and write an expression that describes Cheryl's total earnings in terms of the variables.

Let c represent the number of children's lessons.

Let a represent the number of adult's lessons

Total Earnings: $20c + 30a + 125$

b) One winter, Cheryl gave eight children's lessons and six adult lessons. What were her total earnings?

$20c + 30a + 125$

Sub in $c=8$, $a=6$

$20(8) + 30(6) + 125$

$= 160 + 180 + 125$

$= 465$

$$\begin{array}{r} 160 \\ 180 \\ +125 \\ \hline 465 \end{array}$$

∴ she earned $465 for the winter.

Adding and Subtracting Polynomials

Like Terms have the exact same variable parts. The exponents on all variables must be identical for terms to be like.

Circle identify

Ex. 1. ~~Identify~~ the like terms in each group and ~~circle~~ their coefficients.

a) $2x, -3x^2, -x$
c:2 c:−1

b) $4m^2, 2m, -3m^2$
c:4 c:−3

c) $ab^2, 4a^2b, -2b^2a$
$-2ab^2$
c:1 C=−2

Rule: When simplifying polynomial expressions without brackets, only collect (add or subtract) "like" terms.

Ex. 2. Simplify.

a) $2x - 3 + 5x + 1$

$= +2x + 5x - 3 + 1$

$= 7x - 2$

b) $-1x - 1x - 1x + 1x$

$= -3x + 1x$

$= -2x$

c) $\frac{3}{5}a - \frac{1}{2}b - \frac{1}{5}a + \frac{3}{4}b$

$= \frac{3}{5}a - \frac{1}{5}a - \frac{1}{2}b + \frac{3}{4}b$

$= +\frac{3}{5}a - \frac{1}{5}a - \frac{2}{4}b + \frac{3}{4}b$

$= \frac{2}{5}a + \frac{1}{4}b$

d) $-3u + 2 - u^2 - 5 + 3u + 2u^2$

$= -1u^2 + 2u^2 - 3u + 3u + 2 - 5$

$= 1u^2 + 0u - 3$

$= u^2 - 3$

Ex. 3. Simplify to a single power by using the exponent laws.

a) $2^{3x} \times 2^{4x} \div 2^{-x}$

$= 2^{3x+4x-(-x)}$

$= 2^{3x+4x+1x}$

$= 2^{8x}$

b) $3^{2ab} \div 3^{3a^2} \cdot 3^{-4ba}$

$= 3^{2ab-3a^2+(-4ab)}$

$= 3^{2ab-3a^2-4ab}$

$= 3^{-3a^2+2ab-4ab}$

$= 3^{-3a^2-2ab}$

Rule: When adding or subtracting polynomials in brackets multiply every term in the brackets by $+1$ *or* -1.

Ex. 4. Simplify.

a) $+1(-5p^2-2p)+1(3p^2-5p)$

$=-5p^2-2p+3p^2-5p$

$=-5p^2+3p^2-2p-5p$

$=-2p^2-7p$

b) $+1(3x^2y+5xy-9)+1(2x^2y-4xy+4)$

$=3x^2y+5xy-9+2x^2y-4xy+4$

$=+3x^2y+2x^2y+5xy-4xy-9+4$

$=5x^2y+xy-5$

c) $+1(x+y-z)-1(x-y+z)$

$=x+y-z-x+y-z$

$=x-x+y+y-z-z$

$=0x+2y-2z$

$=2y-2z$

d) $+1(5y^2-5y+1)-1(-2y^2-3)$

$=5y^2-5y+1+2y^2+3$

$=5y^2+2y^2-5y+1+3$

$=7y^2-5y+4$

e) $+1(3x^2-5x+1)-1(2x^2-3)+1(x^2-3x)$

$=3x^2-5x+1-2x^2+3+x^2-3x$

$=3x^2-2x^2+x^2-5x-3x+1+3$

$=2x^2-8x+4$

f) $\left(\frac{3}{4}x-\frac{2}{3}y\right)-1\left(-\frac{1}{2}x+\frac{5}{7}y\right)$

$=\frac{3}{4}x-\frac{2}{3}y+\frac{1}{2}x-\frac{5}{7}y$

$=\frac{3}{4}x+\frac{1}{2}x-\frac{2}{3}y-\frac{5}{7}y$

$=+\frac{3}{4}x+\frac{2}{4}x-\frac{14}{21}y-\frac{15}{21}y$

$=\frac{5}{4}x-\frac{29}{21}y$ * no mixed

Ex. 5. Simplify to a single power by using the exponent laws and evaluate if possible.

a) $\frac{2^{(3x-8)}}{2^{(5x-4)}}$

$=2^{(3x-8)-1(5x-4)}$

$=2^{3x-8-5x+4}$

$=2^{3x-5x-8+4}$

$=2^{-2x-4}$

b) $3^{(7y^2-8y)}\times3^{(-3y^2-5y)}\div3^{(4y^2-13y)}$

$=3^{(7y^2-8y)+(-3y^2-5y)-(4y^2-13y)}$

$=3^{7y^2-8y-3y^2-5y-4y^2+13y}$

$=3^{7y^2-3y^2-4y^2-8y-5y+13y}$

$=3^0$

$=1$

Multiplying a Polynomial by a Monomial

Algebraic expressions can be simplified using the **Distributive Property**. The term in front of the brackets is ***"shared"*** with every term inside the brackets through multiplication**.**

Ex. 1. Simplify the following expressions by first multiplying, then collecting "like" terms when possible.

a) $2(3x-1)$

$= 6x - 2$

b) $-5(3x^2 - 2y + 4)$

$= -15x^2 + 10y - 20$

c) $(3-6w)(-2)$

$= -2(3-6w)$

$= -6 + 12w$

$= 12w - 6$

d) $+\frac{5}{7}\left(\frac{3}{4}x - \frac{1}{2}y + \frac{2}{3}\right)$

$= \frac{5}{7}\left(\frac{3}{4}x\right) - \frac{5}{7}\left(\frac{1}{2}y\right) + \frac{5}{7}\left(\frac{2}{3}\right)$

$= \frac{15}{28}x - \frac{5}{14}y + \frac{10}{21}$

e) $\frac{1}{2}(2w-6) - \frac{2}{3}(9w-6)$

$= \frac{1}{\cancel{2}_1}\cdot\frac{\cancel{2}^1}{1}w - \frac{1}{\cancel{2}_1}\cdot\frac{\cancel{6}^3}{1} - \frac{2}{\cancel{3}_1}\cdot\frac{\cancel{9}^3}{1}w + \frac{2}{\cancel{3}_1}\cdot\frac{\cancel{6}^2}{1}$

$= w - 3 - 6w + 4$

$= w - 6w - 3 + 4$

$= -5w + 1$

f) $18 - 2(x-3) + 4x$

$= 18 - 2x + 6 + 4x$

$= -2x + 4x + 18 + 6$

$= 2x + 24$

g) $3[2 + 5(2k-1)]$

$= 3[2 + 10k - 5]$

$= 3[10k - 3]$

$= 30k - 9$

Note: $a^m \cdot a^n = a^{m+n}$

 $x^1 \cdot x^1 = x^2$ vs $1x + 1x = 2x$ Date:____________________

Multiplying a Polynomial by a Monomial (Continued)

Ex. 1. Expand and simplify.

a) $-2y(3y+1)$

$= -6y^2 - 2y$

b) $-3n^3(3n+1-2n^2)$

$= -9n^4 - 3n^3 + 6n^5$

c) $4x(3x^2-2x+1)-3x(2x^2-5)$

$= 12x^3 - 8x^2 + 4x - 6x^3 + 15x$

$= 12x^3 - 6x^3 - 8x^2 + 4x + 15x$

$= 6x^3 - 8x^2 + 19x$

d) $3x(4x-5y)-2y(2x+3y)$

$= 12x^2 - 15xy - 4xy - 6y^2$

$= 12x^2 - 19xy - 6y^2$

e) $2x-3x[5-1(2x-1)]$

$= 2x - 3x[5 - 2x + 1]$

$= 2x - 3x[-2x + 6]$

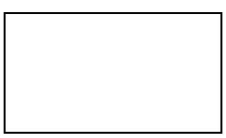
$= 2x + 6x^2 - 18x$

$= 6x^2 - 16x$

fully multiplied & like terms combined.

Ex. 2. a) Find a ***simplified*** expression for the area and perimeter of the rectangle shown.

b) Evaluate the perimeter and the area if $x = 6\ cm$.



$w = 2x\ cm$

$l = (3x+5)\ cm$

a) $A = l \times w$

$A = w \times l$

$A = 2x(3x+5)$

$A = 6x^2 + 10x$

$\therefore A = (6x^2+10x)\ cm^2$

$P = 2l + 2w$

$P = 2(3x+5) + 2(2x)$

$P = 6x + 10 + 4x$

$P = 10x + 10$

$\therefore P = (10x + 10)\ cm$

b) If $x = 6\ cm$

$A = 6(6)^2 + 10(6)$

$A = 216 + 60$

$\therefore A = 276\ cm^2$

If $x = 6$

$P = 10(6) + 10$

$P = 60 + 10$

$\therefore P = 70\ cm.$

Multiplying a Binomial by a Binomial or Trinomial

Recall: Algebraic expressions can be simplified using the **Distributive Property**. The term in front of the brackets is ***"shared"*** with every term inside the brackets through multiplication**.**
Using the **Distributive Property**, each term of the binomial in the first set of brackets is multiplied with each term of the binomial or trinomial in the second set of brackets.

Ex. 1. Simplify each of the following by expanding and then adding like terms.

a) $(n+5)(n-6)$

$= n^2 - 6n + 5n - 30$

$= n^2 - n - 30$

b) $(2n-3)(n-4)$

$= 2n^2 - 8n - 3n + 12$

$= 2n^2 - 11n + 12$

c) $(4x-5)(6x+7)$

$= 24x^2 + 28x - 30x - 35$

$= 24x^2 - 2x - 35$

d) $(3x-5y)^2$

* no exp. laws
for add/subtract

$= (3x-5y)(3x-5y)$

$= 9x^2 - 15xy - 15xy + 25y^2$

$= 9x^2 - 30xy + 25y^2$

e) $(2t'+1)(4t^2-2t+1)$

$= 8t^3 - 4t^2 + 2t + 4t^2 - 2t + 1$

$= 8t^3 - 4t^2 + 4t^2 + 2t - 2t + 1$

$= 8t^3 + 1$

f) $(3t+5)(t^2-3t-8)$

$= 3t^3 - 9t^2 - 24t + 5t^2 - 15t - 40$

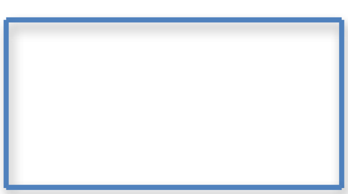
$= 3t^3 - 9t^2 + 5t^2 - 24t - 15t - 40$

$= 3t^3 - 4t^2 - 39t - 40$

Ex. 2. Find an expression for the area of the rectangle below.

$l = (2x+3y)$

$w = (x+2y)$



$A = l \times w$

$A = (2x+3y)(x+2y)$

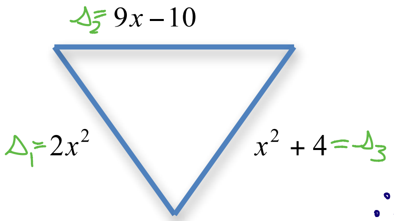
$A = 2x^2 + 4xy + 3xy + 6y^2$

$A = (2x^2 + 7xy + 6y^2)$ units^2

Geometry Extension: Worksheet #2- *Mystery Message*

Part 1. Find the perimeter.

O



$P = a + b + c$

$P = s_1 + s_2 + s_3$

$P = 1(2x^2) + 1(9x - 10) + 1(x^2 + 4)$

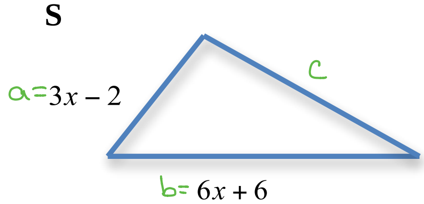
$P = 9x - 10 + 2x^2 + x^2 + 4$

$P = 2x^2 + x^2 + 9x - 10 + 4$

$\therefore P = (3x^2 + 9x - 6)$ units

Part 2. Find the missing side length. The perimeter, P, is given.

S



$P = 13x + 11$

$c = P - a - b$

$c = P - (a + b)$

$c = (13x + 11) - [(3x - 2) + 1(6x + 6)]$

$c = 13x + 11 - (3x - 2 + 6x + 6)$

$c = 13x + 11 - (3x + 6x - 2 + 6)$

$c = 13x + 11 - 1(9x + 4)$

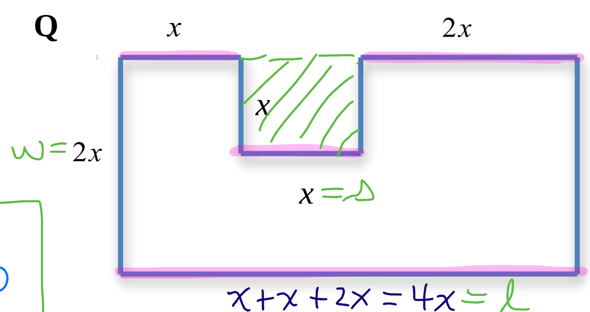
$c = 13x + 11 - 9x - 4$

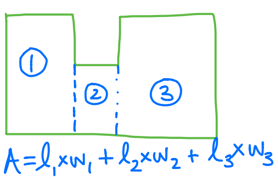
$c = 13x - 9x + 11 - 4$

$\therefore c = (4x + 7)$ units.

Part 3. Find the area.

Q





$A = A_{rectangle} - A_{square}$

$A = l \times w - s^2$

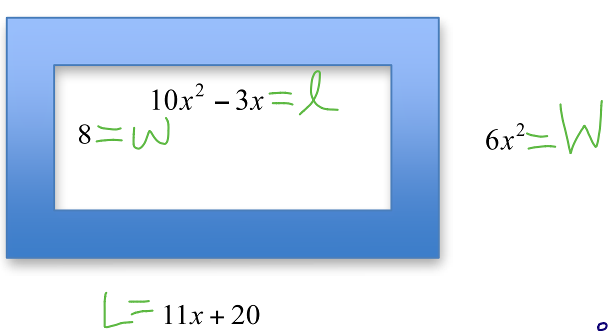
$A = (4x)(2x) - (x)^2$

$A = 8x^2 - x^2$

$\therefore A = (7x^2)$ units^2

Part 4. Find the area of the shaded region.

W



$A = A_{\text{large rectangle}} - A_{\text{small rectangle}}$

$A = L \times W - l \times w$

$A = (6x^2)(11x + 20) - 8(10x^2 - 3x)$

$A = 66x^3 + 120x^2 - 80x^2 + 24x$

$\therefore A = (66x^3 + 40x^2 + 24x)$ units^2

Dividing a Polynomial by a Monomial

$*$ Note: $\frac{a^m}{a^n} = a^{m-n}$

Ex. 1. Simplify each of the following expressions.

a) $\frac{-30x^4y^8z^2}{-6xy^2z^2}$

$= +\frac{30}{6} \cdot \frac{x^4}{x^1} \cdot \frac{y^8}{y^2} \cdot \frac{z^2}{z^2}$

$= 5x^3y^6z^0$

$= 5x^3y^6(1)$

$= 5x^3y^6$

b) $\frac{-4x^2}{-2} - \frac{6x}{-2} + \frac{2}{-2}$

$= 2x^2 + 3x - 1$

c) $\frac{12a^5b^2}{3a^3b^2} + \frac{9a^4b^3}{3a^3b^2} - \frac{6a^3b^4}{3a^3b^2}$

$= 4a^2 + 3ab - 2b^2$

Rule: When dividing by a monomial, **each term** of the polynomial must be divided by the monomial.

Ex. 2. Divide.

a) $\frac{-3x+9}{-3}$

$= \frac{-3x}{-3} + \frac{9}{-3}$

$= x - 3$

b) $\frac{5x-15y+10}{5}$

$= \frac{5x}{5} - \frac{15y}{5} + \frac{10}{5}$

$= x - 3y + 2$

c) $\frac{20x^2y - 30xy^2 + 10xy}{10xy}$

$= \frac{20x^2y}{10xy} - \frac{30xy^2}{10xy} + \frac{10xy}{10xy}$

$= 2x - 3y + 1$

d) $\frac{24xy - 16y}{8y}$

$= \frac{24xy}{8y} - \frac{16y}{8y}$

$= 3x - 2$

e) $\frac{-3a^2b^2 + 5a^3b - 4a^2b}{-a^2b}$

$= \frac{-3a^2b^2}{-a^2b} + \frac{5a^3b}{-a^2b} - \frac{4a^2b}{-a^2b}$

$= 3b - 5a + 4$

Ex. 3. Determine the **Greatest Common Factor (GCF)** of each of the following terms.

a) $8x^2$ *and* $16x$

GCF: $8x$

b) $15abc$ *and* $25bc$

GCF: $5bc$

c) $7x^4y^3z^2$ *and* $2x^3y^2z$

GCF: x^3y^2z

Factoring Polynomials

A **factor** is a number or term that divides evenly into each term of a polynomial. In algebra, **to factor** means to express a polynomial as a **product of its factors**, usually a **monomial** × **polynomial**. The monomial factor is the **G**reatest **C**ommon **F**actor, (**GCF**) and the polynomial factor is the result of dividing each term in the original polynomial by the monomial factor.

Ex. 1. Fill in the missing information.

a) $5x^2 + 35x$

$= 5x(x+7)$

Check: $5x^2 + 35x$ ✓

b) $16x^4y^2 - 8x^2y$

$= 8x^2y(2x^2y - 1)$

check: $16x^4y^2 - 8x^2y$ ✓

Ex. 2. Factor the following polynomials.

***Find the GCF of all terms in the polynomial and then determine the other factor by dividing each term by the GCF.**

a) $10x - 20$

$= 10(x-2)$

Check: $10x - 20$ ✓

b) $22x - 99y$

$= 11(2x - 9y)$

Check: $22x - 99y$ ✓

c) $x^2 - x$

$= x(x-1)$

check: $x^2 - x$

leading negative is in GCF

d) $-m^2 + m$

$= -m(m-1)$

e) $-10x^2 - 25x$

$= -5x(2x+5)$

Check: $-10x^2 - 25x$

f) $13a + 12b - 4c$

∵ GCF = 1

this does not factor.

g) $12x^2y^2z - 8xyz$

$= 4xyz(3xy - 2)$

Check: $12x^2y^2z - 8xyz$ ✓

h) $-16x^2 + 8y + 4x$

$= -4(4x^2 - 2y - x)$

Check: $-16x^2 + 8y + 4x$

i) $35a^4b^3 - 21a^3b + 7a^2b^2$

$= 7a^2b(5a^2b^2 - 3a + b)$

check: $35a^4b^3 - 21a^3b + 7a^2b^2$

Review for Unit 4 Test

Warm-up:

1. Simplify each of the following expressions.

a) $2x(5x^2-2x+7)-3x(4x^2-2)$

$=10x^3-4x^2+14x-12x^3+6x$

$=-2x^3-4x^2+20x$

b) $(2x+5y)(3x-8y)$

$=6x^2-16xy+15xy-40y^2$

$=6x^2-xy-40y^2$

c) $\dfrac{15x^3-10x^2+20x}{5x}$

$=\dfrac{15x^3}{5x^1}-\dfrac{10x^2}{5x^1}+\dfrac{20x^1}{5x^1}$

$=3x^2-2x+4$

first first

d) $\left(\boxed{\dfrac{24xy}{-8x}}\right)-\left(\boxed{\dfrac{-35y^2}{-7y}}\right)$

test

$=-3y-5y$

$=-8y$

2. Factor.

a) $-15x-30y$

$=-15(x+2y)$

b) x^6+x^3

$=x^3(x^3+1)$

c) $-12x^3-6x^2$

$=-6x^2(2x+1)$

d) $20m^5n^2-15m^3n^4+5m^2n^3$

$=5m^2n^2(4m^3-3mn^2+n)$

e) $2^{12}+2^{14}$

$=2^{12}(1+2^2)$

$=2^{12}(5)$

$x^{12}+x^{14}$

$=x^{12}(1+x^2)$

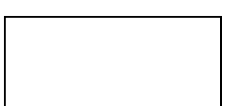
f) $3^{25}-3^{22}$

$=3^{22}(3^3-1)$

$=3^{22}(26)$

extension/bonus

3. Find a simplified expression for the perimeter.



$w=4x$

$l=(3x+5)$

$P=2l+2w$

$P=2l+2w$

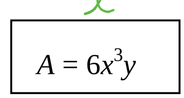
$P=2(3x+5)+2(4x)$

$P=6x+10+8x$

$\therefore P=(14x+10)$ units

4. Find the missing dimension.

$A=l\times w$



l

$A=6x^3y$

$2x=w$

$A=l\times w \rightarrow l=\dfrac{A}{w}$

$l=\dfrac{6x^3y}{2x}$

$\therefore l=3x^2y$ units.

5. Simplify.

a) $\frac{3}{4}(16x-8y)-\frac{1}{3}(9x-3y)$

$= \frac{3}{\cancel{4}_1}\left(\frac{\cancel{16}^4}{1}x\right) - \frac{3}{\cancel{4}_1}\left(\frac{\cancel{8}^2}{1}y\right) - \frac{1}{\cancel{3}_1}\left(\frac{\cancel{9}^3}{1}x\right) + \frac{1}{\cancel{3}_1}\left(\frac{\cancel{3}^1}{1}y\right)$

$= 12x - 6y - 3x + y$

$= 9x - 5y$

b) $\frac{2}{3}\left(\frac{1}{5}x-\frac{7}{8}y+\frac{6}{11}z\right)$

$= \frac{2}{3}\cdot\frac{1}{5}x - \frac{\cancel{2}^1}{3}\cdot\frac{7}{\cancel{8}_4}y + \frac{2}{\cancel{3}_1}\cdot\frac{\cancel{6}^2}{11}z$

$= \frac{2}{15}x - \frac{7}{12}y + \frac{4}{11}z$

6. Simplify and then evaluate for $x=-2$.

$3x-2[2x-4(x-3)]$

$= 3x - 2(2x - 4x + 12)$

$= 3x - 2(-2x + 12)$

$= 3x + 4x - 24$

$= 7x - 24$

If $x = -2$

$= 7(-2) - 24$

$= -14 - 24$

$= -38$

BONUS – Extension Questions

7. Evaluate the following by first factoring the numerator and denominator and then reducing.

*HINT: refer to question #2 e), f) for factoring help.

a) $\frac{5^{20}-5^{18}}{5^{17}+5^{16}}$

$= \frac{5^{18}(5^2-1)}{5^{16}(5^1+1)}$

$= \frac{5^{18}}{5^{16}}\cdot\frac{24}{6}$

$= 5^2\cdot 4$

$= 25\cdot 4$

$= 100$

b) $\frac{2^{99}+2^{102}}{2^{101}-2^{103}}$

$= \frac{2^{99}(1+2^3)}{2^{101}(1-2^2)}$

$= \frac{2^{99}}{2^{101}}\cdot\frac{9}{-3}$

$= 2^{-2}(-3)$

$= \frac{1}{4}\frac{(-3)}{1}$

$= -\frac{3}{4}$

8. Simplify to a single power using the exponent laws, and then evaluate.

a) $4^{2x}\div 4^{x-5}\div 4^{x+8}$

$= 4^{2x-(x-5)-(x+8)}$

$= 4^{2x-x+5-x-8}$

$= 4^{-3}$

$= \frac{1}{64}$

b) $\left(3^{\frac{1}{2}}\right)^{4x-2}\times 3^{5-2x}$

$= 3^{2x-1}\times 3^{5-2x}$

$= 3^{(2x-1)+(5-2x)}$

$= 3^{2x-1+5-2x}$

$= 3^4$

$= 81$

c) $8^x\div 2^{3x-4}$

$= (2^3)^x \div 2^{3x-4}$

$= 2^{3x}\div 2^{3x-4}$

$= 2^{3x-(3x-4)}$

$= 2^{3x-3x+4}$

$= 2^4$

$= 16$