Relations and their Representations

A **relation** is a description of how two quantities are connected. The two quantities that change together are called variable. One quantity will depend on the other and is called the dependent **variable**. The other is the independent **variable** and is the variable whose values you choose.

A **relation** can be represented using one or more of the following:

1. Graph **2.** Table / Ordered Pairs **3.** Equation

4. Words **5.** Picture

In graphing, an ordered pair, (x,y) describes a **point** on the **graph**.

y-coordinate: dependent variable

x-coordinate: independent variable

The data applicable in a relation can be either **discrete** or **continuous**. Discrete data *cannot* be broken into smaller parts so the points on the graph are connected with a **dotted** line. Continuous data *can* be broken down into smaller and smaller parts and still have meaning so the points on the graph are connected with a **solid** line.

countable

measurable

Ex. 1. A plumber charges a flat fee of \$50 plus \$35/ h for a service call.

a) What is the independent variable?

Time (h)

b) What is the dependent variable?

Total Charge (\$)

c) Write an **equation** to represent the plumber's charges in terms of hours of service.

x: Let t represent the time, in hours.

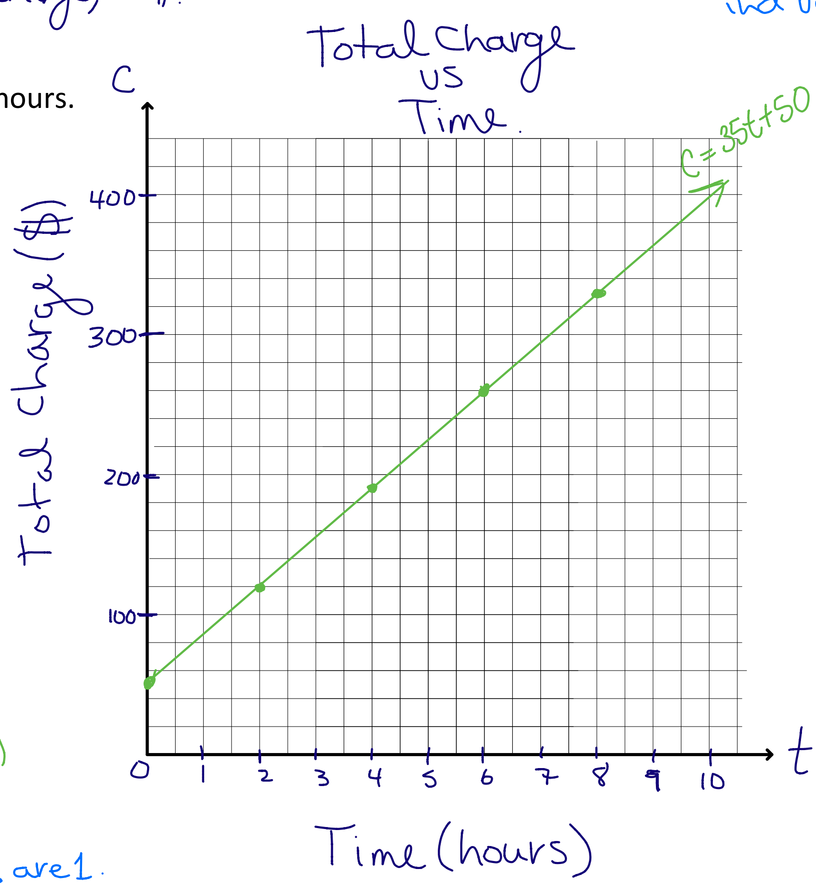
y: Let C represent the total charge, in \$.

$C = 35t + 50$

[* Titles: always "dep var. vs ind var"]

d) Complete a **table of values** for up to 8 hours.

t (ind. var)	C (dep. var) (~5 entries)	
0	35(0)+50 = 50	(0, 50)
2	35(2)+50 = 120	(2, 120)
4	35(4)+50 = 190	(4, 190)
6	35(6)+50 = 260	(6, 260)
8	35(8)+50 = 330	(8, 330)



e) **Graph** the relation for up to 8 hours.

f) Identify this relation as **linear** or **non-linear** with reasons.

Linear:
- Graph is a straight line
- C-values in table increased by the same amount (+70)
- the equation is degree one (first degree equation)
  - ↳ all exp. on all variables are 1.

Ex. 2. Not all relations are linear. Look at the relationship between side length and volume of a cube, where side length is measured in cm and volume is measured in cm^3.

a) What is the independent variable?

Side length (cm)

b) What is the dependent variable?

Volume (cm^3)

c) Write an **equation** to represent the volume of the cube in terms of its side length.

Let x represent the side length, in cm,
Let y represent the volume, in cm^3

$V = lwh$

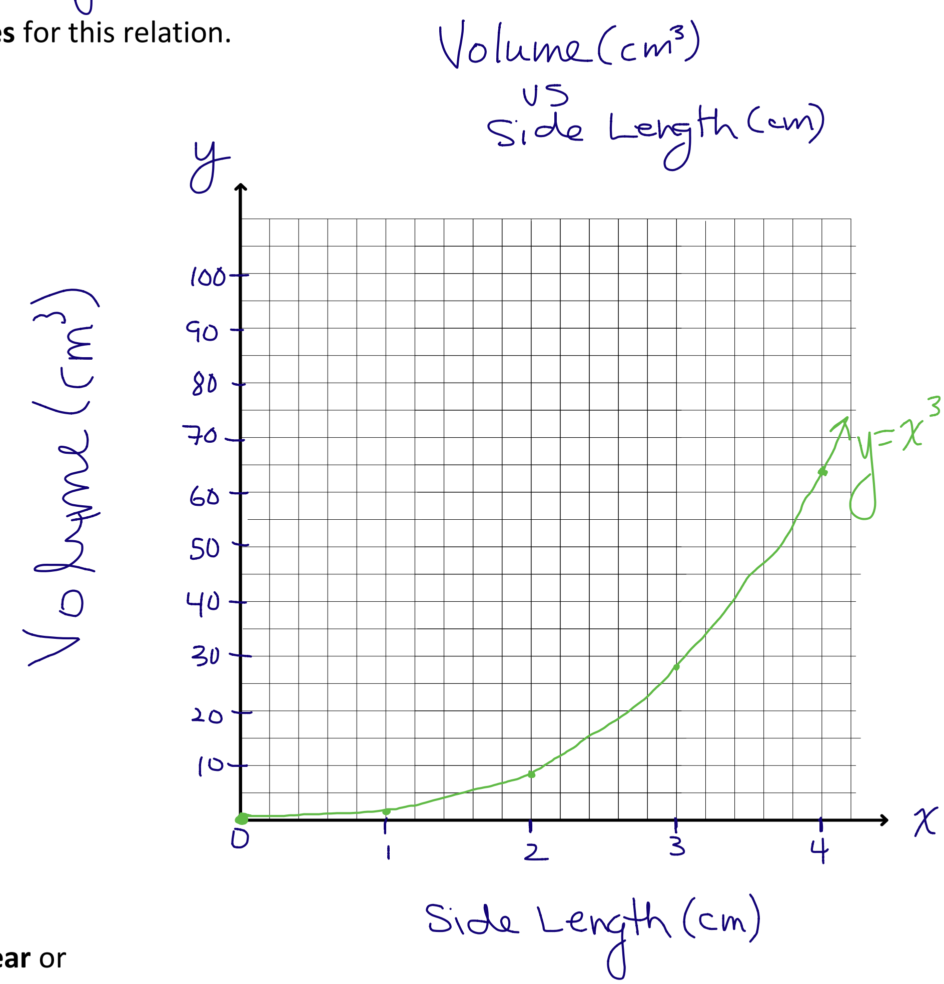
$y = x \cdot x \cdot x$

$y = x^3$

d) Complete a **table of values** for this relation.

x	y
0	0
1	1
2	8
3	27
4	64

e) **Graph** the relation.



f) Identify this relation as **linear** or **non-linear** with reasons.

Non-linear: - graph is not a straight line
- equation is not first degree
- y-values do not change by the same amount

Properties of Linear and Non-Linear Relations

linear

Direct Variation: - is a linear relation in which one variable is a multiple of the other
- the equation looks like $y = mx$
- $(0,0)$ is an ordered pair in the table of values and a point on the graph

Partial Variation: - is a relation in which one variable is a multiple of the other plus a constant amount.
- the equation looks like $y = mx + b$
- $(0,0)$ is not an ordered pair in the table of values or a point on the graph

Ex. 1. A fitness club offers two types of monthly memberships:

- **Membership A**: $4 per visit
- **Membership B**: a flat fee of $12 plus $2 per visit

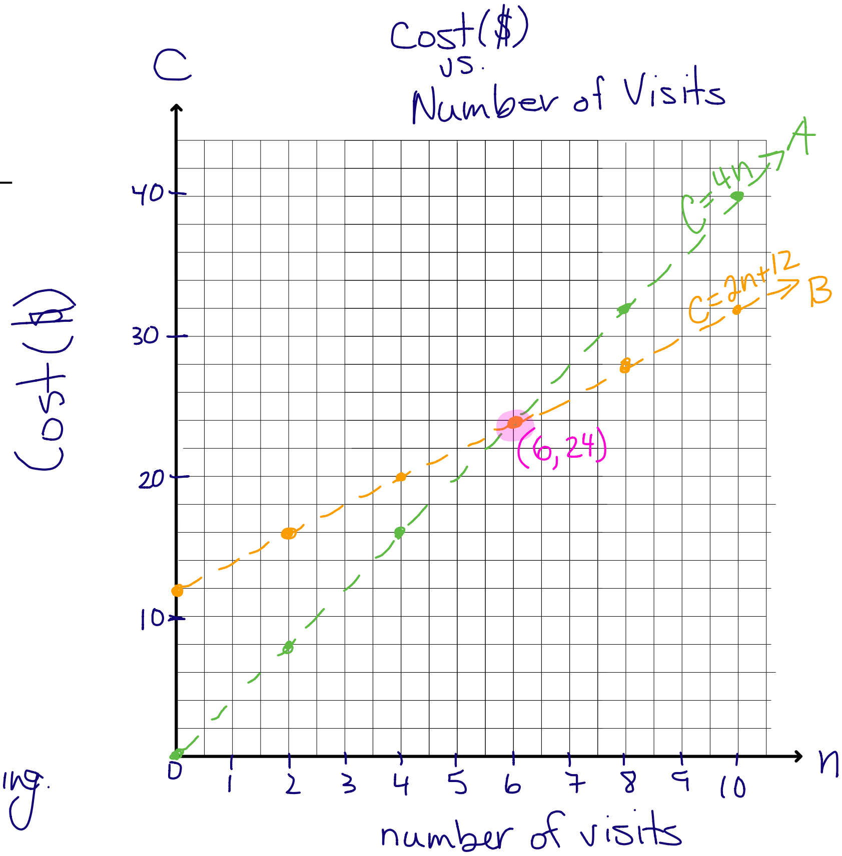
a) Complete a **table of values** for 0 to 10 visits for each relation and then **graph**.

i) Membership A

# of visits n	Total Cost($) C
0	0
2	8
4	16
6	24
8	32
10	40

ii) Membership B

# of visits n	Total Cost($) C
0	12
2	16
4	20
6	24
8	28
10	32



b) Identify this set of data as **discrete** or **continuous** with reasons.

This data is discrete because $\frac{1}{2}$ a visit has no meaning.

c) Write an equation relating the cost, C, in $ and the number of visits, n, for each membership.

Membership A: $C=4n$ Membership B: $C=2n+12$

d) Use the equations to determine the number of visits for which both monthly costs are the same.

$$4n=2n+12$$
$$4n-2n=12$$
$$\frac{2n}{2}=\frac{12}{2}$$
$$n=6$$

$\therefore$ at 6 visits the cost is the same.

e) Identify each relation as **direct** or **partial variation** with reasons.

Membership A is direct variation:
- $(0,0)$ is in the table/point on graph
- equation is in the form $y=mx$ $(C=4n)$

Membership B is partial variation:
- $(0,0)$ is not in table/point on graph
- equation is in the form $y=mx+b$ $(C=2n+12)$

Linear Relation: - equation is of the first degree, the graph is a straight line, and in a **table of values** the **first differences** are **constant**.

Non-linear Relation: - equation is not of the first degree, the graph is not a straight line, and in a **table of values** the **first differences** are **not constant**.

The **first differences**, Δy, "delta y", represent the differences between consecutive y- values where the differences between consecutive x- values is constant.

Ex. 2. a) Petri Dish A: Bacteria can double every 30 minutes in a growth medium. Starting with one bacterial cell, set up a table of values for the growth of this bacterial colony over 2 hours and use **first differences** to determine if the relation is linear or non-linear.

b) Petri Dish B: Bacteria are growing more slowly due to an antibiotic in the growth medium. Starting with one bacterial cell, only 5 new cells are generated every 40 minutes. Set up a table of values for the growth of this bacterial colony over 2 hours and use **first differences** to determine if the relation is linear or non-linear.

Let x represent the time in minutes and let y represent the number of bacteria in the colony.

Petri Dish A

a)

Time (min) x	# of bacteria y	Δy
0	1 $\quad y_1$	
30	2 $\quad y_2$	$y_2-y_1=2-1=1$
60	4 $\quad y_3$	$y_3-y_2=4-2=2$
90	8 $\quad y_4$	$y_4-y_3=8-4=4$
120	16 $\quad y_5$	$y_5-y_4=16-8=8$

∵ the first differences (Δy) are not constant
∴ this relation is non-linear.

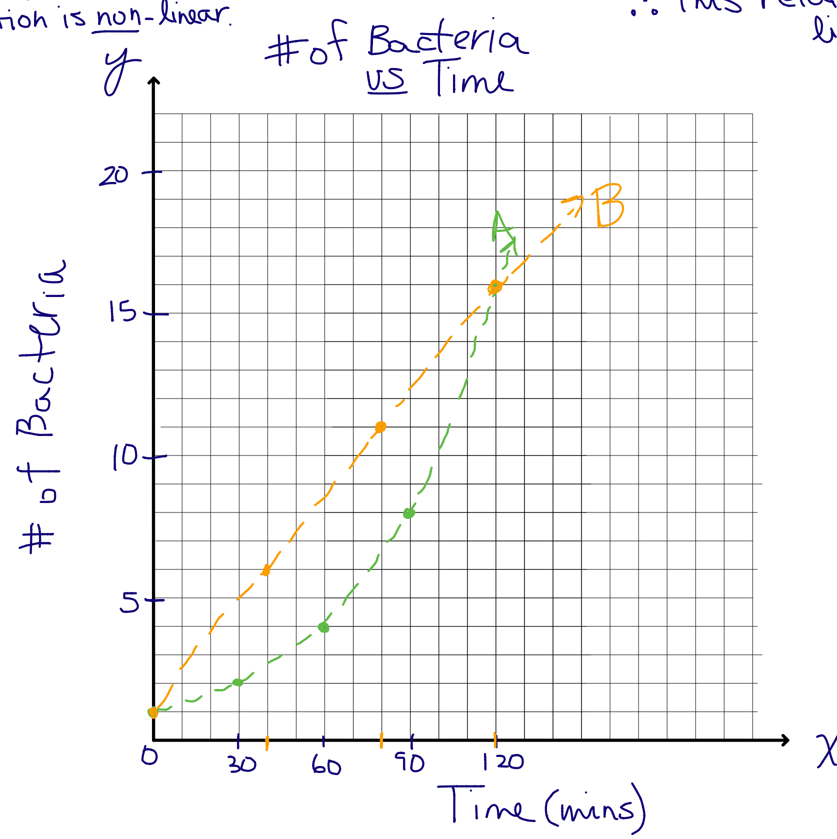
Petri Dish B

b)

Time (min) x	# of bacteria y	Δy
0	1 $\quad y_1$	
40	6 $\quad y_2$	$y_2-y_1=6-1=5$
80	11 $\quad y_3$	$y_3-y_2=11-6=5$
120	16 $\quad y_4$	$y_4-y_3=16-11=5$

∵ the first differences (Δy) are constant
∴ This relation is linear.

c) Graph both relations.



Coordinate Geometry in 4 Quadrants

To describe the location of points on a plane we use the Cartesian Coordinate System.

Definitions:

x-axis - the horizontal number line which extends left and right.

y-axis - the vertical number line which extends up and down.

origin - the point (0, 0) where the axes meet
x y

ordered pair - a point of the form (x, y) located on a Cartesian plane

x-coordinate - the first number in an ordered pair describing the horizontal position of the point.

y-coordinate - the second number in an ordered pair describing the vertical position of the point.

quadrant - the 4 regions created by the x-axis & y-axis

in Roman numerals I, II, III, IV

Example: Plot the following points on the grid below: A(4,3) B(-5,1) C(-2,-4) D(0,5) E(-2,0) F(6,1) G(-1,4) H($1\frac{1}{2}$,−2) I(-$3\frac{1}{2}$,$5\frac{1}{2}$) O(0,0)

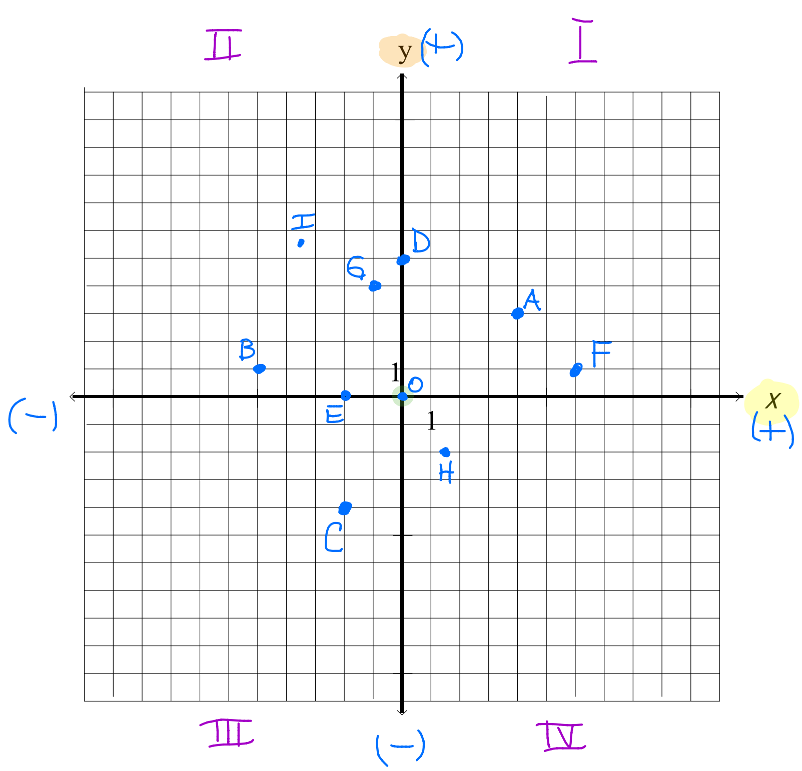
Which points are in:

Quadrant I
A, F

Quadrant II
B, I, G

Quadrant III
C

Quadrant IV
H



Describe in which quadrant ordered pairs have the following signs.

(+ , +) I

(+ , -) IV

(- , +) II

(- , -) III

on x-axis: E, O
on y-axis: D, O

Using a Table of Values to Graph Linear Relations

A linear equation can be written in the form $y = mx + b$ **where** m and b are numbers.

Ex. 1. Determine if the following points are on the line $y = -3x + 7$: **A** $(1, 4)$ **B** $(-2, 14)$

Check: A $x = 1$, $y = 4$

$LS = y$
$= 4$

$RS = -3x + 7$
$= -3(1) + 7$
$= -3 + 7$
$= 4$

$\because LS = RS$

$\therefore A(1,4)$ is on the line $y = -3x + 7$.

Check: B $x = -2$, $y = 14$

$LS = y$
$= 14$

$RS = -3x + 7$
$= -3(-2) + 7$
$= 6 + 7$
$= 13$

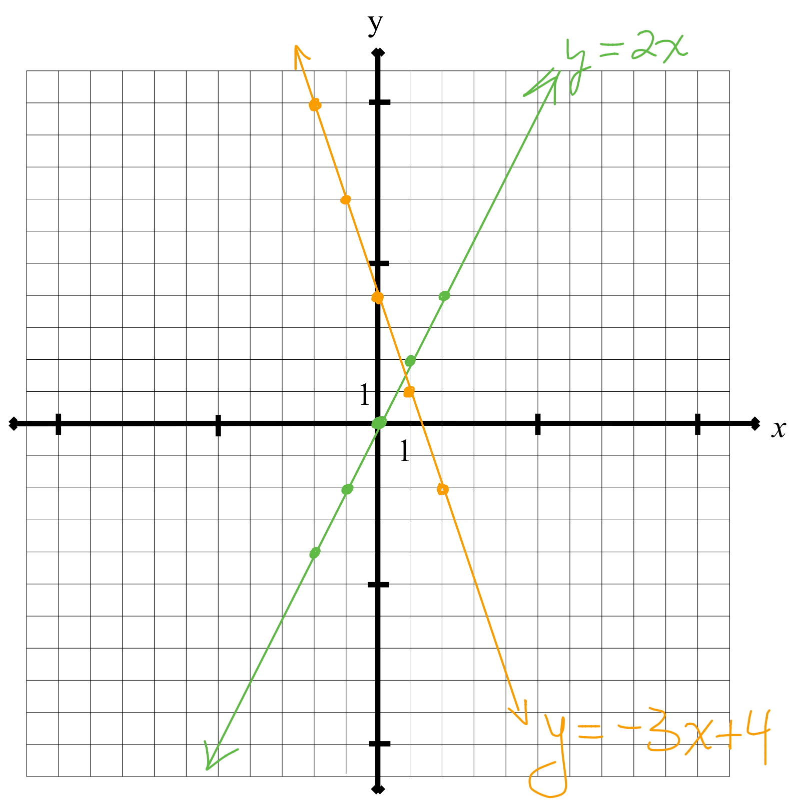
$\because LS \neq RS$

$\therefore B(-2, 14)$ is not on the line $y = -3x + 7$

Ex. 2. Graph each relation by making a **Table of Values** with five entries.

a) $y = 2x$

x	$y = 2x$	
-2	$2(-2) = -4$	$(-2,-4)$
-1	$2(-1) = -2$	$(-1,-2)$
0	$2(0) = 0$	$(0,0)$
1	$2(1) = 2$	$(1,2)$
2	$2(2) = 4$	$(2,4)$

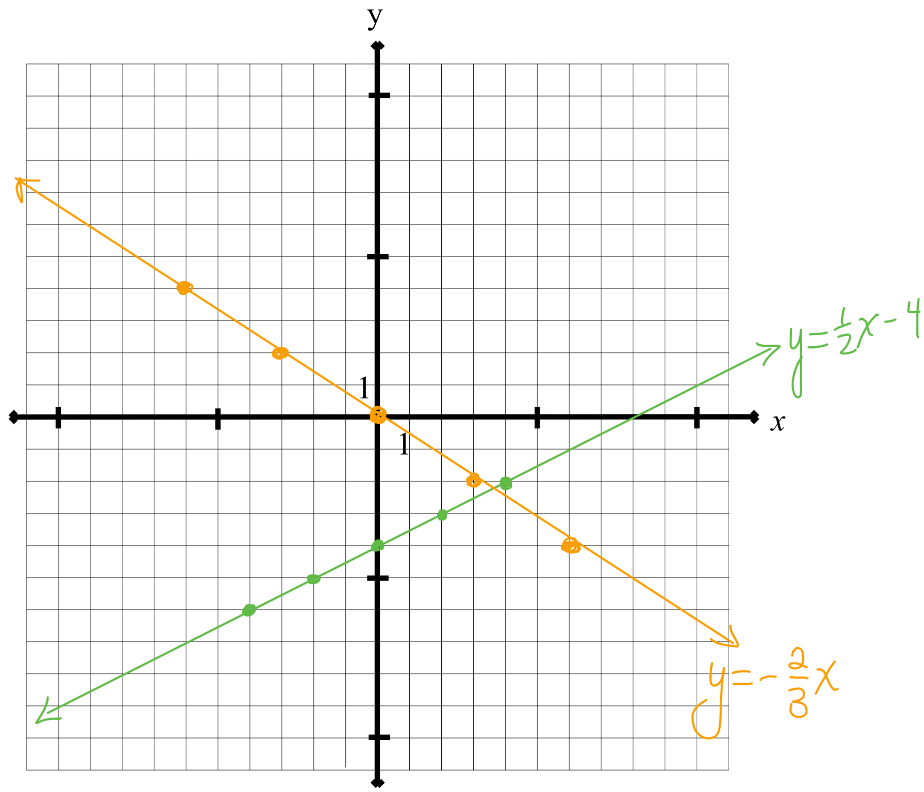


b) $y = -3x + 4$

x	$y = -3x + 4$
-2	$-3(-2) + 4 = 10$
-1	$-3(-1) + 4 = 7$
0	$-3(0) + 4 = 4$
1	$-3(1) + 4 = 1$
2	$-3(2) + 4 = -2$

c) $y = \frac{1}{2}x - 4$

	x	$y = \frac{1}{2}x - 4$
-2×2	-4	$\frac{1}{2}(-4) - 4 = -2 - 4 = -6$
-1×2	-2	$\frac{1}{2}(-2) - 4 = -1 - 4 = -5$
0×2	0	$\frac{1}{2}(0) - 4 = 0 - 4 = -4$
1×2	2	$\frac{1}{2}(2) - 4 = 1 - 4 = -3$
2×2	4	$\frac{1}{2}(4) - 4 = 2 - 4 = -2$

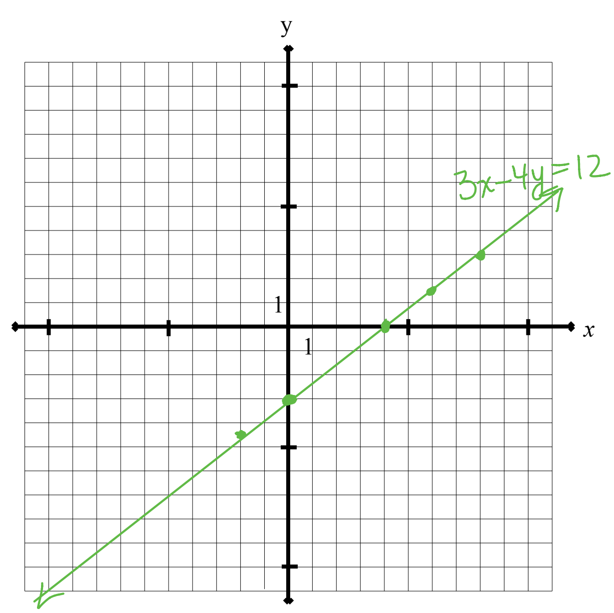


d) $y = -\frac{2}{3}x$

	x	$y = -\frac{2}{3}x$
-2×3	-6	$-\frac{2}{3}(-6) = 4$
-1×3	-3	$-\frac{2}{3}(-3) = 2$
0×3	0	$-\frac{2}{3}(0) = 0$
1×3	3	$-\frac{2}{3}(3) = -2$
2×3	6	$-\frac{2}{3}(6) = -4$

Ex. 3. Complete the following table of values and graph.

	3x - 4y = 12	
	x	**y**
1	0	-3
2	-2	$-4\frac{1}{2}$
3	8	3
4	6	$\frac{3}{2} = 1\frac{1}{2}$
5	4	0



(1) If $x = 0$:
$3(0) - 4y = 12$
$0 - 4y = 12$
$\frac{-4y}{-4} = \frac{12}{-4}$
$y = -3$

(2): If $x = -2$
$3(-2) - 4y = 12$
$-6 - 4y = 12$
$\frac{-4y}{-4} = \frac{18}{-4}$
$y = -\frac{9}{2}$
$y = -4\frac{1}{2}$

(3) If $y = 3$:
$3x - 4(3) = 12$
$3x - 12 = 12$
$3x = 12 + 12$
$\frac{3x}{3} = \frac{24}{3}$
$x = 8$

(4) If $x = 6$:
$3(6) - 4y = 12$
$18 - 4y = 12$
$\frac{-4y}{-4} = \frac{-6}{-4}$
$y = \frac{3}{2}$

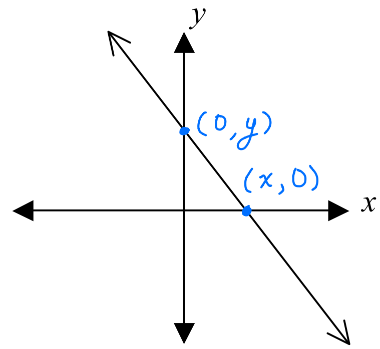
(5) If $y = 0$:
$3x - 4(0) = 12$
$3x - 0 = 12$
$\frac{3x}{3} = \frac{12}{3}$
$x = 4$

Using Intercepts to Graph Linear Relations

Definitions:

The x **-intercept** is the x value of the point where the line crosses the x-axis. To find the x **-intercept**, let y=0 and then solve for x.

The y **-intercept** is the y value of the point where the line crosses the y-axis. To find the y **-intercept**, let x=0 and then solve for y.



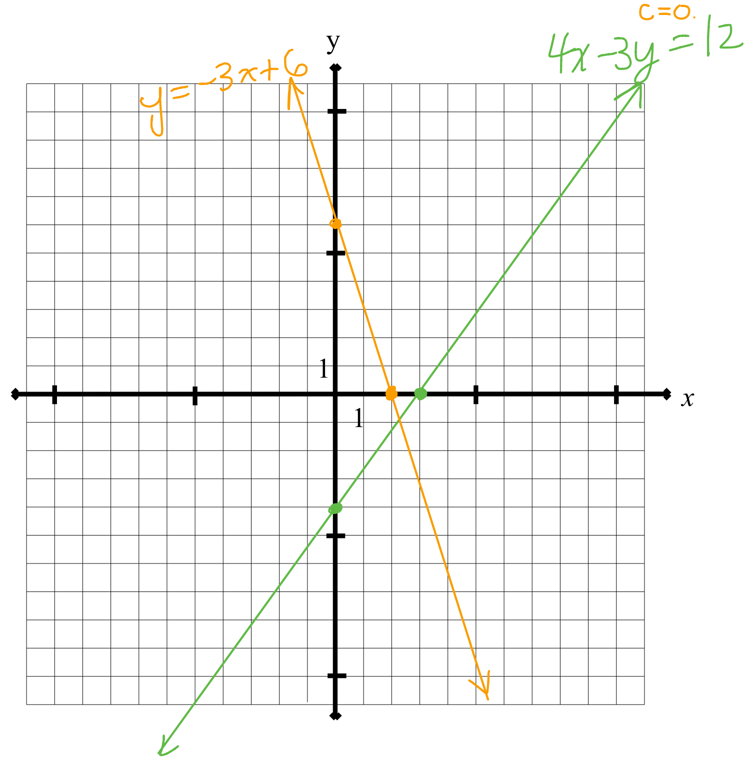
$Ax + By = C$
or $Ax + By + C = 0$ } good form for this method.
*best if A & B divide evenly into C.
*does not work if C=0.

Ex. 1. Graph the following linear relations by finding the intercepts.

a) $4x - 3y = 12$

For x-int:
Let y=0
$4x - 3(0) = 12$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$
∴ x-intercept is 3
plot: (3, 0)

For y-int:
Let x=0
$4(0) - 3y = 12$
$\frac{-3y}{-3} = \frac{12}{-3}$
$y = -4$
∴ the y-intercept is -4
plot: (0, -4)



$y = mx + b$

b) $y = -3x + 6$

For x-int:
Let y=0
$0 = -3x + 6$
$\frac{3x}{3} = \frac{6}{3}$
$x = 2$
∴ x-intercept is 2
Plot (2, 0)

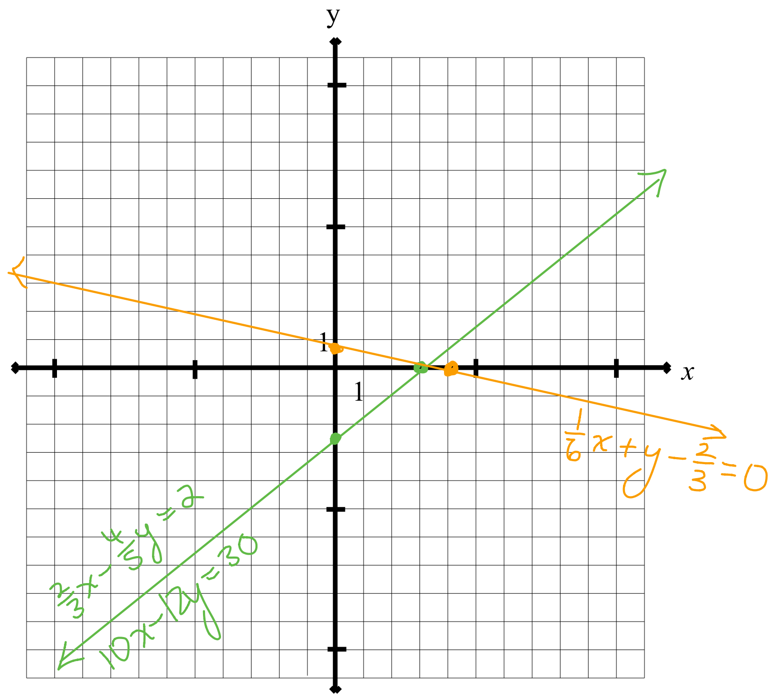
For y-int:
Let x=0
$y = -3(0) + 6$
$y = 6$
∴ y-intercept is 6
Plot (0, 6)

Ex. 2. Graph the following linear relations by finding the intercepts.

a) $\frac{2}{3}x - \frac{4}{5}y = 2$ LCD = 15

$\frac{\cancel{15}^{5}}{1}\left(\frac{2}{\cancel{3}_{1}}x\right) - \frac{\cancel{15}^{3}}{1}\left(\frac{4}{\cancel{5}_{1}}y\right) = 15(2)$

$10x - 12y = 30$



For x-int:

Let $y = 0$

$10x - 12(0) = 30$

$\frac{10x}{10} = \frac{30}{10}$

$x = 3$

$\therefore$ x-intercept is 3

plot $(3, 0)$

For y-int:

Let $x = 0$

$10(0) - 12y = 30$

$\frac{-12y}{-12} = \frac{30}{-12}$

$y = -\frac{5}{2}$

$\therefore$ y-intercept is $-\frac{5}{2}$

plot $(0, -2\frac{1}{2})$

b) $\frac{1}{6}x + y - \frac{2}{3} = 0$

For x-int:

Let $y = 0$

$\frac{1}{6}x + (0) - \frac{2}{3} = 0$

$\frac{1}{6}x - \frac{2}{3} = 0 \longrightarrow \cancel{6}^{1}\left(\frac{1}{\cancel{6}_{1}}x\right) - \cancel{6}^{2}\left(\frac{2}{\cancel{3}_{1}}\right) = 6(0)$

$x - 4 = 0$

$x = 4$

$\frac{1}{6}x = \frac{2}{3}$

$x = \frac{2}{3} \div \frac{1}{6}$

$x = \frac{2}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{1}$

$x = 4$

$\therefore$ x-intercept is 4

plot $(4, 0)$

For y-int:

Let $x = 0$

$\frac{1}{6}(0) + y - \frac{2}{3} = 0$

$y - \frac{2}{3} = 0$

$y = \frac{2}{3}$

$\therefore$ y-intercept is $\frac{2}{3}$

plot $(0, \frac{2}{3})$

Using the Slope and y-intercept to Graph Linear Relations

WARMUP

Graph each of the following linear relations using the indicated method.

table of values

$y = \frac{3}{5}x + 2$

x	y
-10	-4
-5	-1
0	2
5	5
10	8

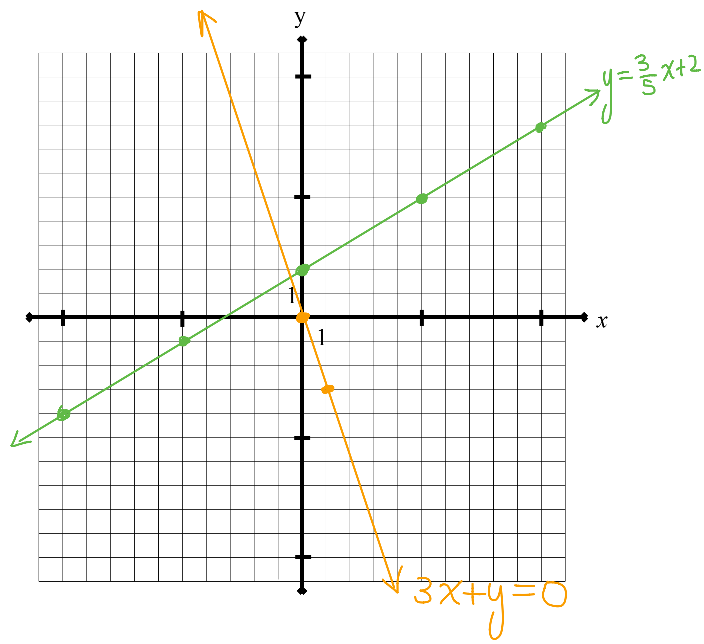
intercepts

$3x + y = 0 \leftarrow y = -3x$

For x-int: let $y=0$
$3x + (0) = 0$
$\frac{3x}{3} = \frac{0}{3}$
$x = 0$
Plot: $(0,0)$

For y-int: let $x=0$
$3(0) + y = 0$
$y = 0$
Plot: $(0,0)$

*need one more point
Let $x=1$: $3(1) + y = 0$
$3 + y = 0$
$y = -3$
Plot $(1,-3)$



The **rate of change** of the linear relation is called the Slope of the line.

The ***y*** value of the point where the line crosses the y-axis is called the y-intercept.

For $y = \frac{3}{5}x + 2$ the slope is $\frac{3}{5}$ and the y-intercept is 2.

For $y = -3x$ the slope is -3 and the y-intercept is 0.

Summary:

The graph of the relation $\boxed{y = mx + b}$ is a straight line with slope m and y-intercept b.

Slope measures the steepness of the line & direction.

Slope = $\frac{\text{rise}}{\text{run}}$ "rise over run"

The greater the magnitude of slope m (ignoring the sign) the steeper the line.

A line with a **positive slope** rises to the right.

A line with a **negative slope** falls to the right.

Note: The equation of a line in **slope, y-intercept form** is in the form $y = mx + b$.

Ex. 1. Complete the following table.

Equation	Slope	y-intercept
$y = 2x - 5$	2	−5
$y = 1x + 10$	1	10
$y = -5x$	−5	0
$y = \frac{2}{3}x - 1$	$\frac{2}{3}$	−1
$y = 3$	0	3

$y = 0x + 3$

*horizontal line

Ex. 2. Write the equation of the line in **slope, y-intercept form** with:

a) $m = \frac{1}{2}$ and $b = -1$

$y = \frac{1}{2}x - 1$

b) $m = 7$ and $b = 0$

$y = 7x + 0$ or $y = 7x$

Ex. 3. Use the **slope and y-intercept** to graph the following lines:

a) $y = 3x - 5$

① $m = 3 = \frac{3}{1}\ \frac{\text{rise}}{\text{run}}$

$b = -5$

Plot $(0, -5)$

① State m & b's values (integer values over 1)

② plot the y-int (b)

③ From the y-int point, use the slope to rise ↑ or ↓ then run → to the next point. Put a dot.

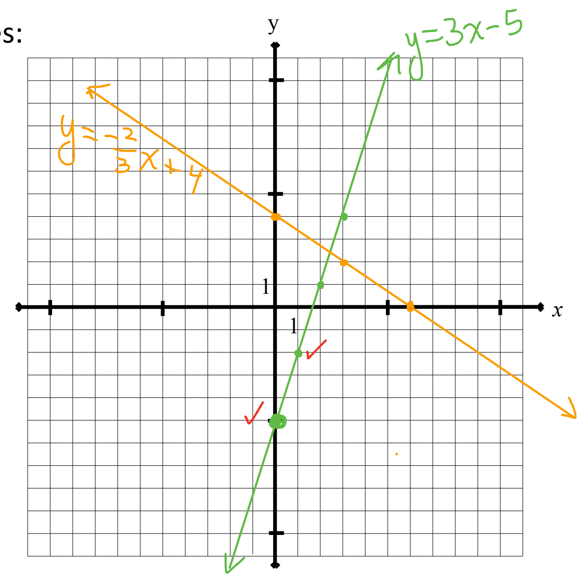
④ Repeat ③ if needed.

b) $y = -\frac{2}{3}x + 4$

$m = \frac{-2}{3}\ \frac{\text{rise}\downarrow}{\text{run}\rightarrow}$

$b = 4$

Plot $(0, 4)$



Ex. 4. Rewrite each of the following equations in **slope, y-intercept form** and graph. $y = mx + b$

a) $5x + 2y = 0$

$\frac{2y}{2} = \frac{-5x}{2}$

$y = \frac{-5}{2}x$

$m = \frac{-5}{2}\ \frac{\text{rise}}{\text{run}}$

$b = 0$ Plot $(0, 0)$

b) $x - 4y - 28 = 0$

$\frac{-4y}{-4} = \frac{-1x}{-4} + \frac{28}{-4}$

$y = \frac{1}{4}x - 7$

$m = \frac{1}{4}\ \frac{\text{rise}}{\text{run}}$

$b = -7$ Plot $(0, -7)$