Solving Pairs of Linear Equations by Graphing

When solving a **system of linear relations**, the **point of intersection** of their graphs is the solution to that system of linear equations.

Ex. 1. Solve the following systems of linear equations graphically by determining the point of intersection, if possible.

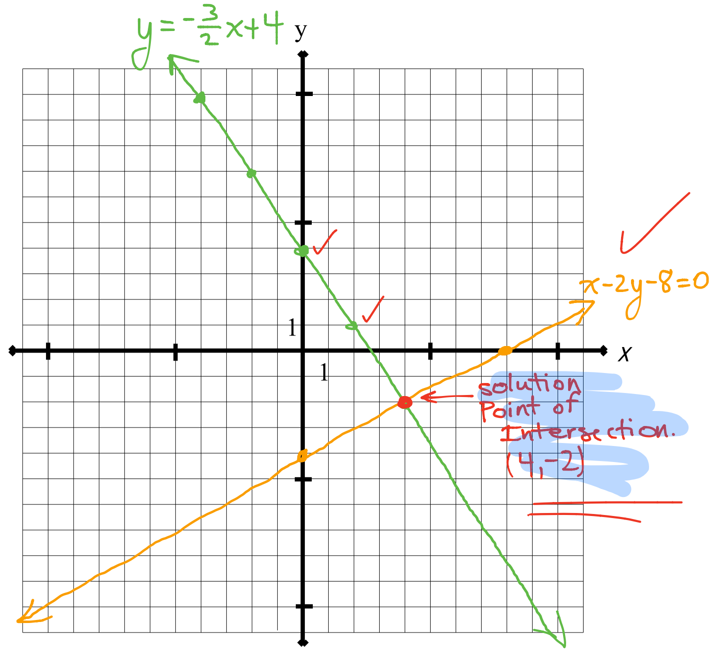
a) $y = -\frac{3}{2}x + 4$ ①
$x - 2y - 8 = 0$ ②

① $m = -\frac{3}{2}$
$b = 4$
plot $(0,4)$

② For x-int:
Let $y = 0$
$x - 2(0) - 8 = 0$
$x - 8 = 0$
$x = 8$
∴ plot $(8,0)$

For y-int:
Let $x = 0$
$0 - 2y - 8 = 0$
$-2y = 8$
$y = -4$
∴ plot $(0,-4)$

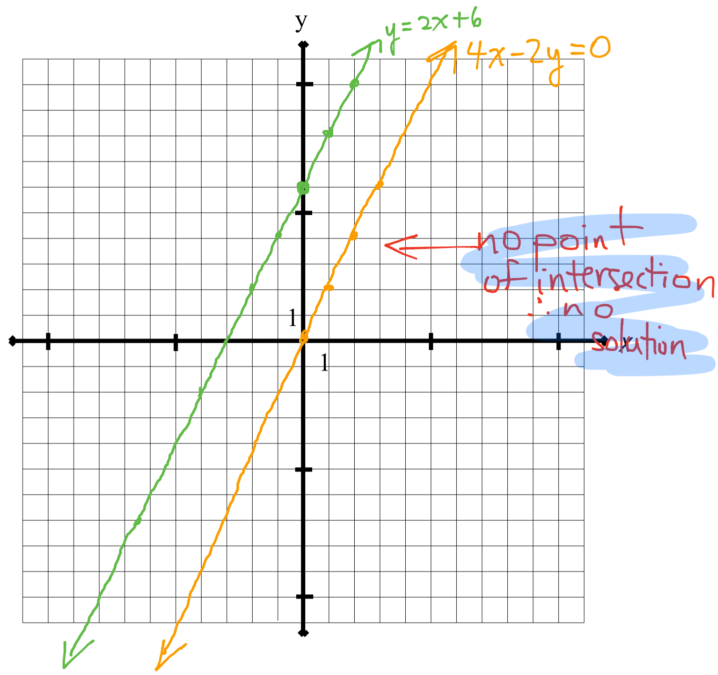
∴ the solution (x,y) is the point $(4,-2)$



b) $y = 2x + 6$ ①
$4x - 2y = 0$
$\frac{-2y}{-2} = \frac{-4x}{-2}$
$y = 2x$ ②

① $m = 2 = \frac{2}{1}$
$b = 6$, plot $(0,6)$

② $m = 2 = \frac{2}{1}$
$b = 0$ plot $(0,0)$



Ex. 1. Without graphing determine if the point $(-1, -3)$ is the solution to the following linear system:

$y = 4x + 1$ ①
$x - y = 5$ ②

LS/RS check
with $x = -1, y = -3$

① $LS = y$ $RS = 4x + 1$
$LS = -3$ $RS = 4(-1) + 1$
$= -4 + 1$
$= -3$
$\boxed{LS = RS}$

② $LS = x - y$ $RS = 5$
$LS = (-1) - (-3)$
$LS = -1 + 3$
$LS = 2$
$\boxed{LS \neq RS}$

∴ $LS \neq RS$ for both equations
∴ $(-1,-3)$ is not the solution.

Solving Pairs of Linear Equations Algebraically

Recall:
When solving a **system of linear relations**, the **point of intersection** of their graphs is the solution to that system of linear equations.

Ex. 1. Solve the following system of linear equations **a) graphically** **b) algebraically**

a) $y = -2x + 8$ ①
$3x - 2y - 6 = 0$ ②

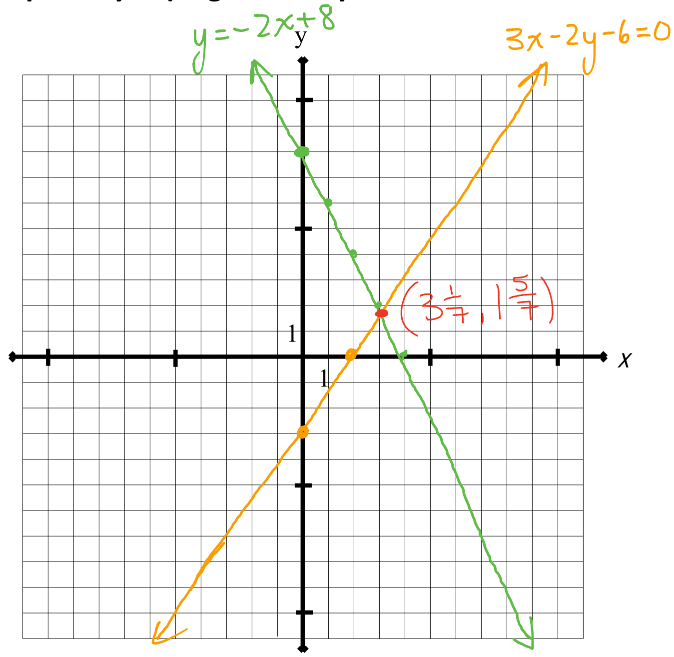
① $y = -2x + 8$:
$m = -2$, $b = 8$
or $m = \frac{-2}{1}$
Plot $(0,8)$

② $3x - 2y - 6 = 0$:

For x-int:
Let $y=0$
$3x - 2(0) - 6 = 0$
$3x - 6 = 0$
$\frac{3x}{3} = \frac{6}{3}$
$x = 2$
Plot $(2,0)$

For y-int:
Let $x=0$
$3(0) - 2y - 6 = 0$
$-2y - 6 = 0$
$\frac{-2y}{-2} = \frac{6}{-2}$
$y = -3$
Plot $(0,-3)$

Point of intersection: $(3,2)$?
① LS $= y$ RS $= -2x+8$
$= 2$ $= -2(3)+8$
$= -6+8$
$= 2$
LS=RS
② LS $= 3x-2y-6$ RS $= 0$
$= 3(3)-2(2)-6$
$= 9-4-6$
$= -1$
LS$\neq$RS
$\therefore (3,2)$ is not the point of intersection



b) $y = -2x + 8$ ①
$3x - 2y - 6 = 0$ ②

Rearrange ② to $y = mx + b$ form.
$3x - 2y - 6 = 0$
$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{6}{-2}$
$y = \frac{3}{2}x - 3$ ③

Compare ① and ③:
Set $y = y$
$-2x + 8 = \frac{3}{2}x - 3$
$2(-2x) + 2(8) = 2\left(\frac{3}{2}x\right) - 2(3)$
$-4x + 16 = 3x - 6$
$-4x - 3x = -6 - 16$
$\frac{-7x}{-7} = \frac{-22}{-7}$
$x = \frac{22}{7}$
(or $x = 3\frac{1}{7}$)

Sub $x = \frac{22}{7}$ into ①
$y = \frac{-2}{1}\left(\frac{22}{7}\right) + \frac{8}{1}$
$y = \frac{-44}{7} + \frac{56}{7}$
$y = \frac{12}{7}$
(or $y = 1\frac{5}{7}$)

$\therefore$ the solution is $\left(\frac{22}{7}, \frac{12}{7}\right)$
or $\left(3\frac{1}{7}, 1\frac{5}{7}\right)$

Steps for Solving a Linear System of Equations Algebraically Using the Method of Comparison

1. Rewrite each equation in $y = mx + b$ (**slope, y-intercept**) form.
2. Compare the first equation to the second equation and then solve for x.
3. Substitute the x value found in step **2.** into either equation in step **1.** to find the y value.

Ex. 2. Solve the following systems of linear equations **algebraically** using **comparison**.

a) $2x-3y=12 \rightarrow \frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3} \rightarrow y = \frac{2}{3}x - 4$ ①

$x = 4y+1 \rightarrow \frac{-4y}{-4} = \frac{-x}{-4} + \frac{1}{-4} \rightarrow y = \frac{1}{4}x - \frac{1}{4}$ ②

Compare ① to ②:

$$\frac{2}{3}x - 4 = \frac{1}{4}x - \frac{1}{4}$$

$$\frac{\cancel{12}^{4}}{1}\left(\frac{2}{\cancel{3}_{1}}x\right) - 12(4) = \frac{\cancel{12}^{3}}{1}\left(\frac{1}{\cancel{4}_{1}}x\right) - \frac{\cancel{12}^{3}}{1}\left(\frac{1}{\cancel{4}_{1}}\right)$$

$$8x - 48 = 3x - 3$$

$$8x - 3x = -3 + 48$$

$$\frac{5x}{5} = \frac{45}{5}$$

$$x = 9$$

Sub $x=9$ into ①

$$y = \frac{2}{\cancel{3}_{1}}\frac{(\cancel{9}^{3})}{1} - 4$$

$$y = 6 - 4$$

$$y = 2$$

$\therefore$ the solution is $(9, 2)$.

b) $x - y = 3 \rightarrow \frac{-y}{-1} = \frac{-x}{-1} + \frac{3}{-1} \rightarrow \boxed{y = x - 3}$ ①

$6x + 4y = 13 \rightarrow \frac{4y}{4} = \frac{-6x}{4} + \frac{13}{4} \rightarrow \boxed{y = -\frac{3}{2}x + \frac{13}{4}}$ ②

} important skill

Compare ① to ②:

$$x - 3 = -\frac{3}{2}x + \frac{13}{4}$$

$$4(x) - 4(3) = \frac{\cancel{4}^{2}}{1}\left(\frac{-3}{\cancel{2}_{1}}x\right) + \frac{\cancel{4}^{1}}{1}\left(\frac{13}{\cancel{4}_{1}}\right)$$

$$4x - 12 = -6x + 13$$

$$4x + 6x = 13 + 12$$

$$10x = 25$$

$$x = \frac{25}{10}$$

$$x = \frac{5}{2}$$

$$\left(\text{or } x = 2\tfrac{1}{2}\right)$$

Sub $x = \frac{5}{2}$ into ①:

$$y = \left(\frac{5}{2}\right) - \frac{3}{1}$$

$$y = \frac{5}{2} - \frac{6}{2}$$

$$y = -\frac{1}{2}$$

$\therefore$ the solution (x, y) is the point $\left(2\tfrac{1}{2}, -\tfrac{1}{2}\right)$

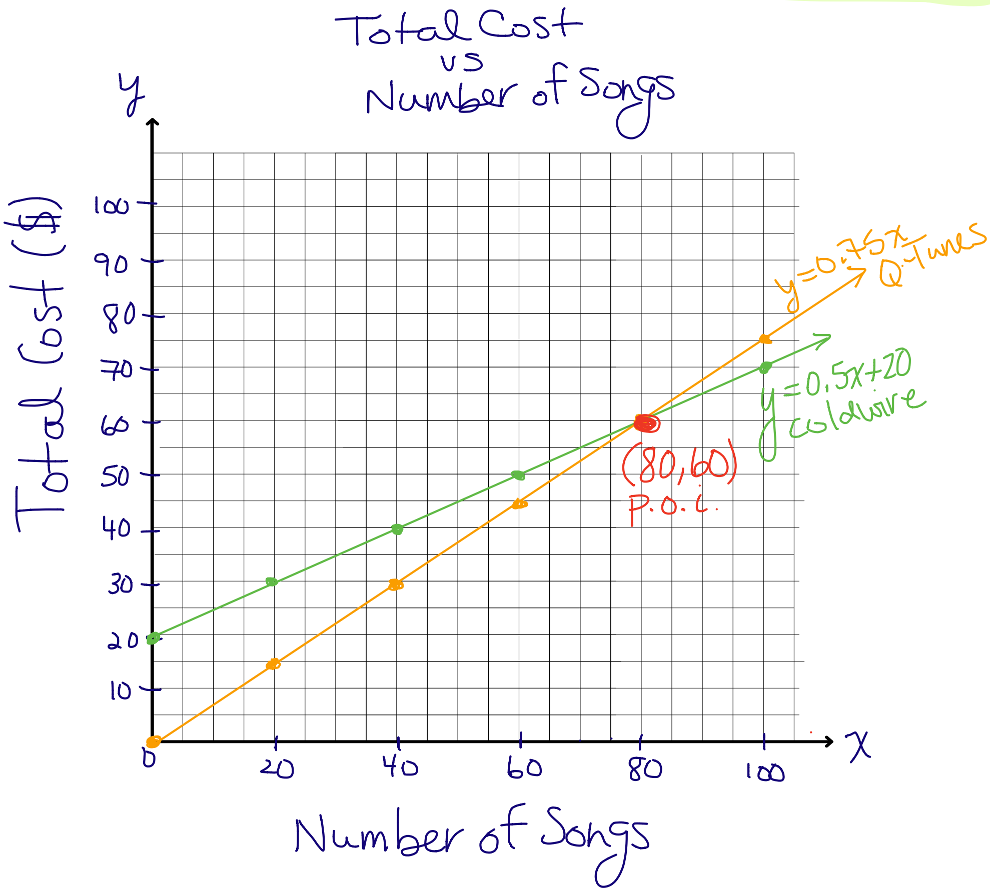
Graphing Applications Part I

Ex. 1. Two music websites provide the latest and the greatest songs from your favourite artists from Bieber to the Beatles. "Coldwire" charges an annual membership fee of $20 per year plus 50¢ for each song downloaded. "Q-tunes" charges 75¢ /song with no membership fee.

a) Write an equation for each website. Include "let" statements".

Let x represent the number of songs downloaded.
Let y represent the total cost, in $.
Coldwire: $y=0.5x+20$ $\left(y=\frac{1}{2}x+20\right)$
Q-Tunes: $y=0.75x$ $\left(y=\frac{3}{4}x\right)$

b) Graph both equations on the same set of axes, for up to 100 songs.



$y=0.5x+20$

x	y
0	20
20	30
40	40
60	50
80	60
100	70

$y=0.75x$

x	y
0	0
20	15
40	30
60	45
80	60
100	75

$\frac{3}{4}\frac{(20)}{1}$

$\frac{3}{4}(40)$

c) Determine the point of intersection. What does the point of intersection mean in this case?

The point of intersection is (80, 60)
This means that both companies charge $60 for 80 songs.

d) What advice would you give to someone who is deciding which website to use?

If you will download more than 80 songs, Coldwire will charge less. For fewer than 80 songs Q-Tunes will charge less.

Ex. 2. The cost to make ice pops is \$0.10 per ice pop, plus \$9.00 in supplies. Each ice pop sells for \$0.40.

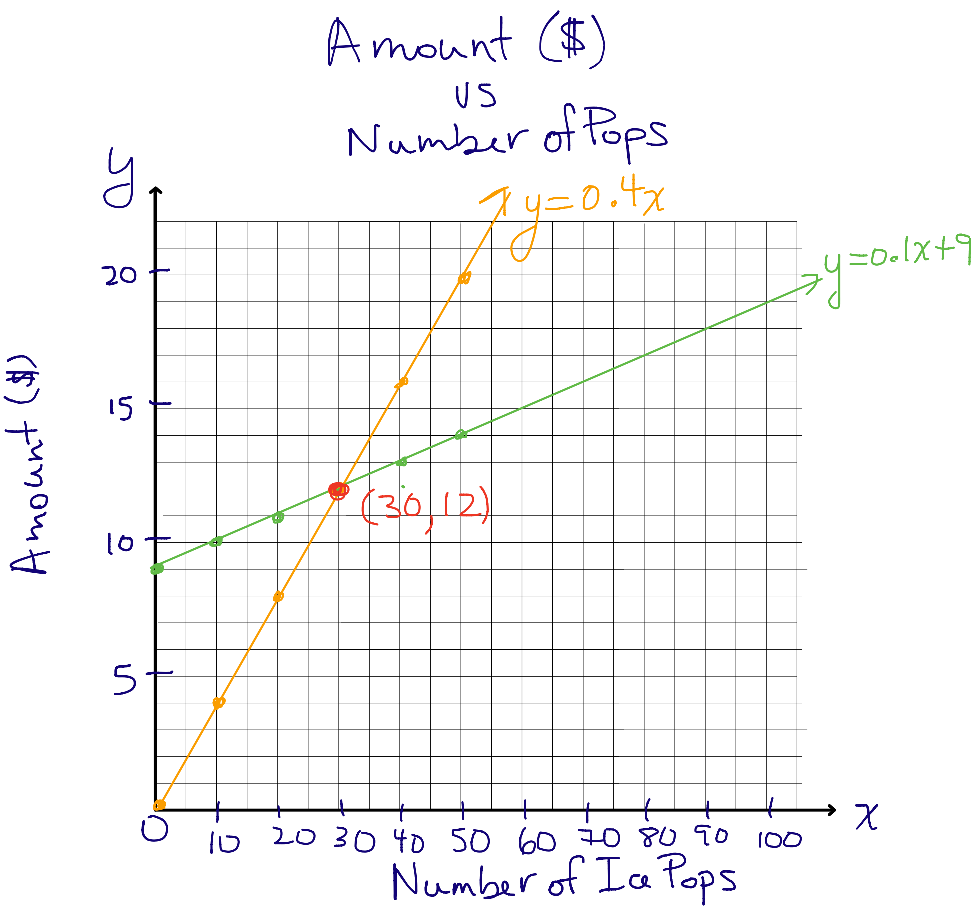
a) Define the variables and then create a system of two linear equations to model this situation.

Let x represent the number of ice pops (made & sold).
Let y represent the amount of money (earned/spent) in \$.

(\$ out) Cost: $y = 0.1x + 9$ ①

(\$ in) Revenue: $y = 0.4x$ ②

b) Graph both equations on the same set of axes, for up to 50 pops.



① Cost $y = 0.1x + 9$

x	y
0	9
10	10
20	11
30	12
40	13
50	14

② Revenue: $y = 0.4x$

x	y
0	0
10	4
20	8
30	12
40	16
50	20

c) How many pops do you need to sell to break even?

∴ To break even (\$ In = \$ out) 30 pops must be sold.

Graphing Applications Part II

Ex. 1. Joe has 26 coins that are all dimes and quarters. The value of the coins is $4.10. How many dimes and how many quarters does Joe have?

a) Define the variables and write equations in terms of your variables for the total number of coins and the total value of the coins.

Let x represent the number of dimes.
Let y represent the number of quarters.

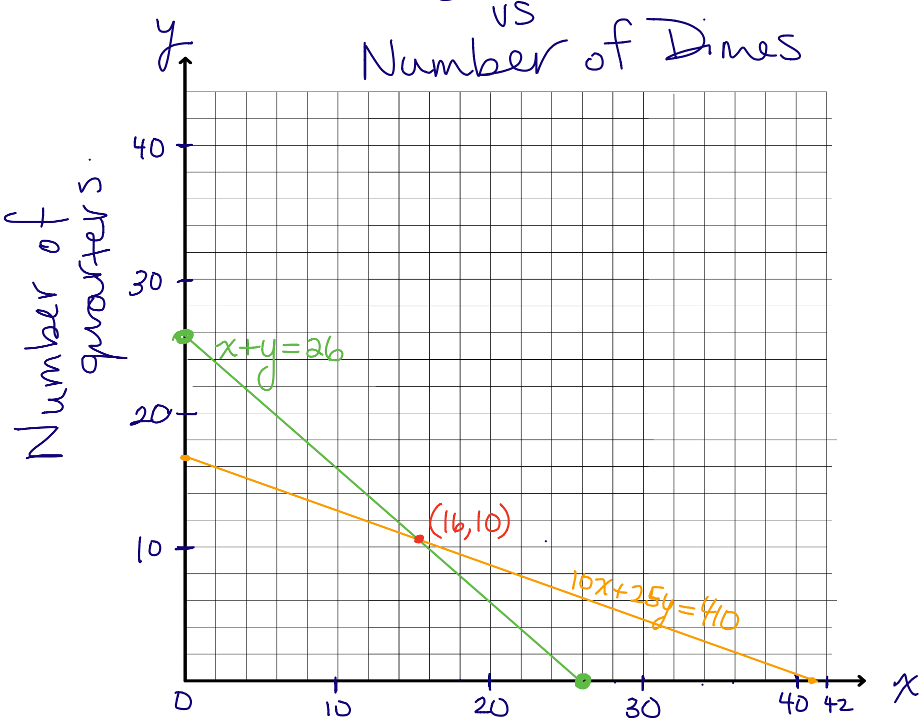
Total Number: $x+y=26$ ①

Total Value: $0.1x+0.25y=4.10$

or $10x+25y=410$ ②

b) Graph both equations on the same set of axes using the **intercept method** for graphing.

Number of Quarters vs Number of Dimes



① $x+y=26$

For x-int: Let $y=0$
$x+0=26$
$x=26$
Plot $(26,0)$

For y-int: Let $x=0$
$0+y=26$
$y=26$
Plot $(0,26)$

② $10x+25y=410$

For x-int: Let $y=0$
$10x+25(0)=410$
$\frac{10x}{10}=\frac{410}{10}$
$x=41$
Plot $(41,0)$

For y-int: Let $x=0$
$10(0)+25y=410$
$\frac{25y}{25}=\frac{410}{25}$
$y=16\frac{10}{25}$
$y=16\frac{2}{5}$
$(y=16.4)$

c) Determine the point of intersection. What does the point of intersection mean in this case?

Test: $(16,10)$

① LS $=x+y$
$=16+10$
$=26$
RS $=26$ ✓

② LS $=10x+25y$
$=10(16)+25(10)$
$=160+250$
$=410$
RS $=410$ ✓

$250 + 160 = 410$

The point of intersection is $(16,10)$

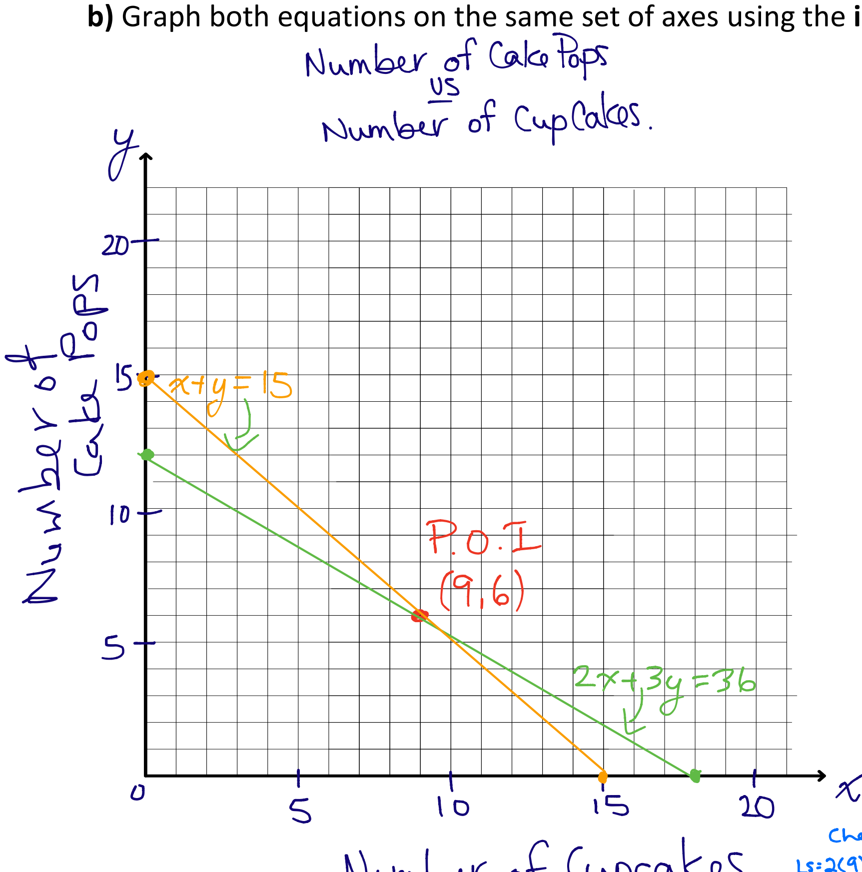
This means that Joe has 16 dimes and 10 quarters.

Ex. 2. The **Cake Box** makes specialty cake pops and cupcakes. Each pop costs \$3 while each cupcake costs \$2. If 15 of these items were ordered at a cost of \$36 determine the number of pops and the number of cupcakes in the order.

a) Define the variables and write equations in terms of your variables for the total number of items in the order and the total cost.

Let x represent the number of cupcakes.
Let y represent the number of cake pops.
Quantity: $x+y=15$ ①
Cost: $2x+3y=36$ ②

b) Graph both equations on the same set of axes using the **intercept method** for graphing.



①
$x+y=15$

For x-int:
Let $y=0$
$x+0=15$
$x=15$
Plot $(15,0)$

For y-int:
Let $x=0$
$0+y=15$
$y=15$
Plot $(0,15)$

②
$2x+3y=36$

For x-int:
Let $y=0$
$2x+3(0)=36$
$2x=36$
$x=18$
Plot $(18,0)$

For y-int:
Let $x=0$
$2(0)+3y=36$
$3y=36$
$y=12$
Plot $(0,12)$

Check:
LS $=2(9)+3(6)$ RS $=36$
$=18+18$ ✓
$=36$

c) Determine the point of intersection. What does the point of intersection mean in this case?

$\therefore$ The point of intersection is $(9,6)$.
This means that 9 cupcakes and 6 cake pops were ordered.