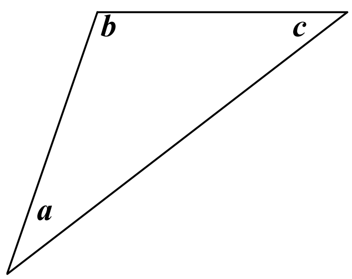
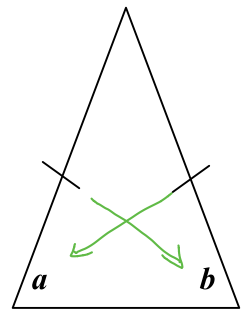
Angle Properties – Part I

1. The **sum** of the **interior angles** of a **triangle** is 180°.

$a+b+c=180^\circ$

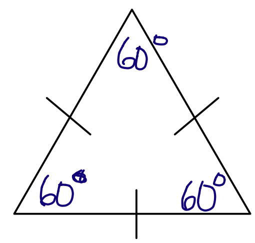


2. In an **isosceles** triangle the two equal angles are opposite the two equal sides.



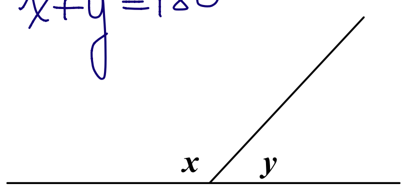
$a=b$

3. In an **equilateral** triangle all side lengths are equal and all angles are equal. Each angle measures 60°.

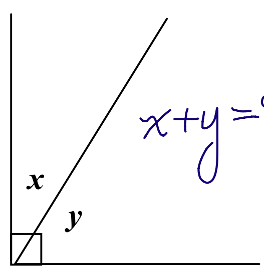


4. Supplementary angles add up to 180°.

$x+y=180^\circ$



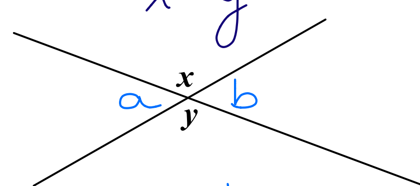
5. Complementary angles add up to 90°.



$x+y=90^\circ$

6. Opposite angles are equal.

$x=y$



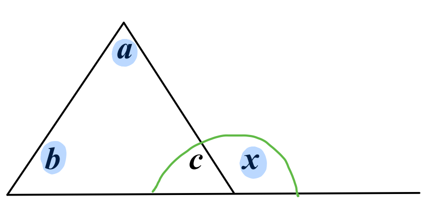
$a=b$

7. The **exterior angle** (x) **of a triangle** is the sum of the two non-adjacent interior angles.

$x=a+b$

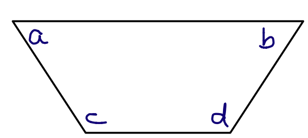
The **exterior angle** of a polygon and the **adjacent interior angle** are **supplementary**.

$c+x=180^\circ$



8. The **sum** of the **interior angles** of a **quadrilateral** is 360°.

$a+b+c+d=360^\circ$



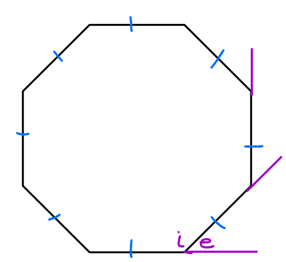
The **sum** of the **interior angles** of a **polygon** with ***n* sides is**

$(n-2)\times 180^\circ$

The **sum** of the **exterior angles** of an ***n*-gon** is 360°.

9. In a **regular polygon** or ***n*-gon**, all **interior angles** are equal and all **exterior angles** are equal.

all sides have equal length



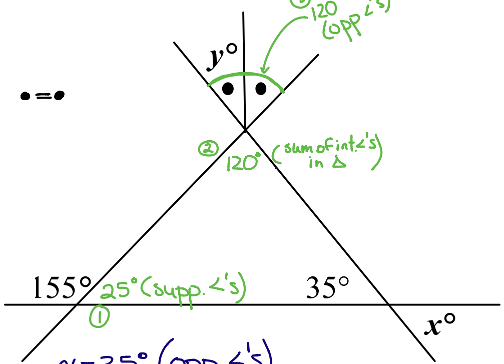
$i=\frac{(n-2)\times 180^\circ}{n}$

$e=\frac{360^\circ}{n}$

$i+e=180^\circ$

Examples: Determine the value of each unknown.

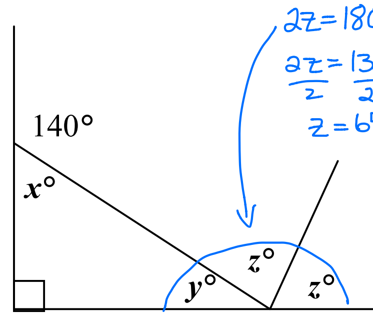
a)



$x = 35°$ (opp. ∠'s)

$y = 60°$ $(120° \div 2)$

b)



$2z = 180 - y$

$\frac{2z}{2} = \frac{130°}{2}$

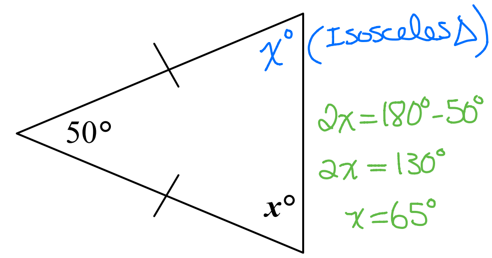
$z = 65°$

$x = 40°$ (Supp. ∠'s)

$y = 50°$ $(y = 180° - (90° + 40°)$ sum of int ∠'s in Δ)

$z = 65°$ (supp. ∠'s)

c)



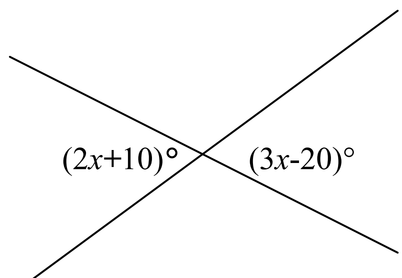
$2x = 180° - 50°$

$2x = 130°$

$x = 65°$

$x = 65°$ (sum of int. ∠'s in Δ)

d)



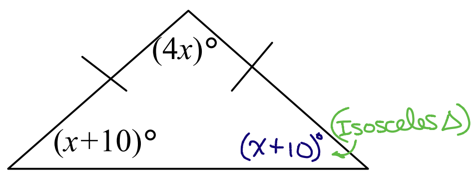
$2x + 10 = 3x - 20$ (opp ∠'s)

$2x - 3x = -20 - 10$

$-x = -30$

$x = 30$

e)



$4x + 2(x+10) = 180$ (Sum of int. ∠'s in Δ)

$4x + 2x + 20 = 180$

$6x + 20 = 180$

$6x = 180 - 20$

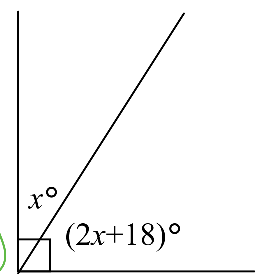
$\frac{6x}{6} = \frac{160}{6}$

$x = \frac{80}{3}$

$x = 26\frac{2}{3}$

$$\begin{array}{r} 26 \\ 3\overline{)80} \\ -6\downarrow \\ 20 \\ -18 \\ 2 \end{array}$$

f)



$x + 2x + 18 = 90$ (Comp. ∠'s)

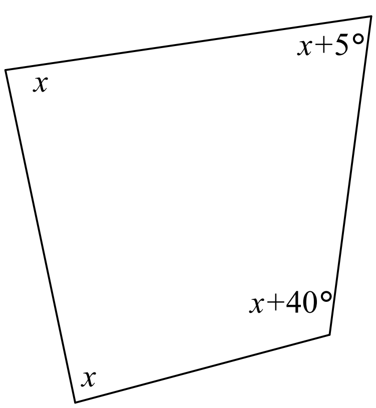
$3x + 18 = 90$

$3x = 90 - 18$

$\frac{3x}{3} = \frac{72}{3}$

$x = 24$

g)



$x + x + x + 40 + x + 5 = 360°$ (Sum of int ∠'s of quadrilateral)

$4x + 45 = 360$

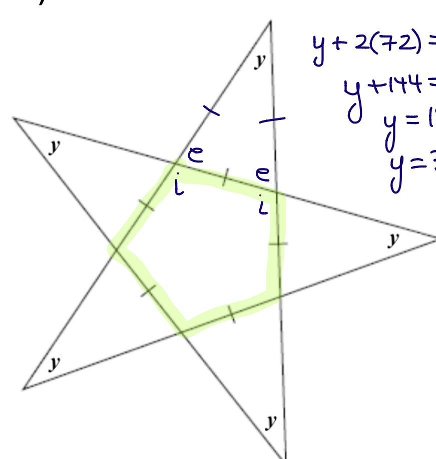
$4x = 360 - 45$

$\frac{4x}{4} = \frac{315}{4}$

$x = 78\frac{3}{4}$

$$\begin{array}{r} 78 \\ 4\overline{)315} \\ -28\downarrow \\ 35 \\ -32 \\ 3 \end{array}$$

h)



$y + 2(72) = 180°$ (sum of int. ∠'s in Δ)

$y + 144 = 180°$

$y = 180 - 144$

$y = 36$

$i = \frac{(5-2) \times 180°}{5}$ (sum of int ∠'s of n-gon $\div n$)

$i = \frac{3 \times 180}{5}$

$i = \frac{540}{5}$

$i = 108°$

$e = 180 - i$ (Supp ∠'s)

$e = 72°$

Angle Properties – Part II

Warm-up:

1. In a regular 11-gon determine the measure of each:

a) exterior angle

$e = \dfrac{360^\circ}{n}$

$e = \dfrac{360^\circ}{11}$

$e = 32\frac{8}{11}^\circ$

$11\overline{)360}$ → 32, -33, 30, -22, 8

b) interior angle

$i = 180^\circ - 32\frac{8}{11}^\circ$ (Supp. ∠'s)

$= 147\frac{3}{11}^\circ$

$180 \times 9 = 1620$

or

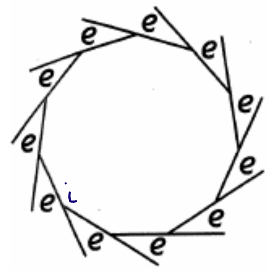
$i = \dfrac{(n-2)\times 180^\circ}{n}$

$= \dfrac{9 \times 180^\circ}{11}$

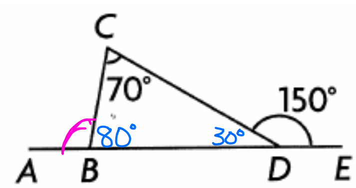
$= \dfrac{1620}{11}$

$= 147\frac{3}{11}^\circ$

$11\overline{)1620}$ → 147, -11, 52, -44, 80, -77, 3



2. Determine the measure of $\angle CBA$ and explain your reasoning.



$\angle CDB = 180^\circ - 150^\circ$ (Supp. ∠)
$= 30^\circ$

$\angle CBD = 180^\circ - 70^\circ - 30^\circ$ (Sum of int. ∠'s in △)
$= 80^\circ$

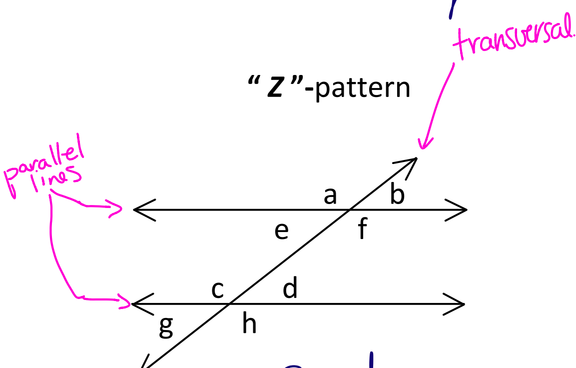
$\angle CBA = 180^\circ - 80^\circ$ (Supp. ∠'s)
$= 100^\circ$

Parallel Lines:

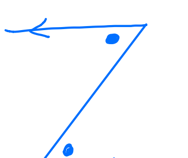
When a **transversal** crosses **parallel lines**, three new classifications of angles are created, aside from *opposite angles* and *supplementary angles*.

1. Alternate angles are equal. **2. Corresponding angles** are equal. **3. Interior angles** add to 180°

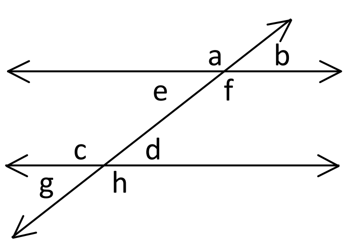
" ***Z*** "-pattern

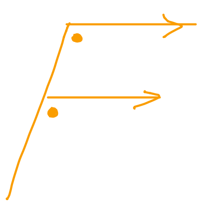


$e = d$
$c = f$



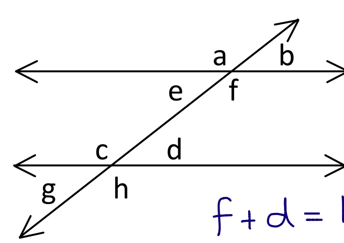
" ***F*** "-pattern



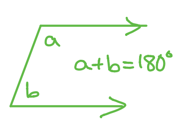


$d = b$
$c = a$
$f = h$
$e = g$

" ***C*** "-pattern



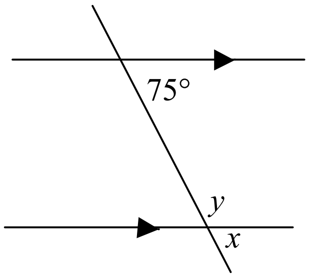
$f + d = 180^\circ$
$c + e = 180^\circ$



Z → alternate angles   F → corresponding angles   C → interior angles

Examples: Determine the value of the unknown variable.

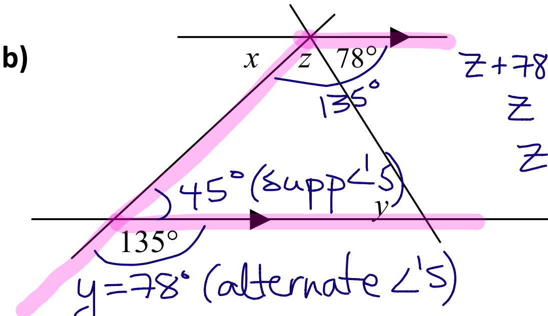
a)



$x = 75°$ (corresponding ∠'s)

$y = 105°$ (interior ∠'s)

b)



$z + 78 = 135$

$z = 135 - 78$

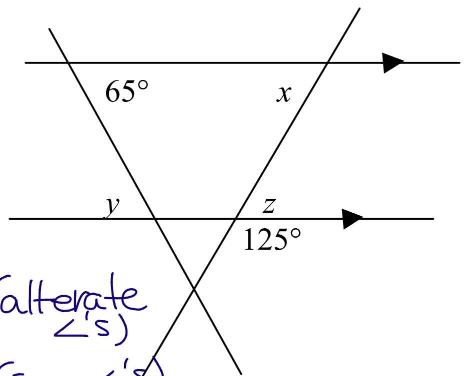
$z = 57$

$y = 78°$ (alternate ∠'s)

$z = 57°$ (corresponding ∠'s)

$x = 45°$ (alt. angles / supp ∠'s)

c)

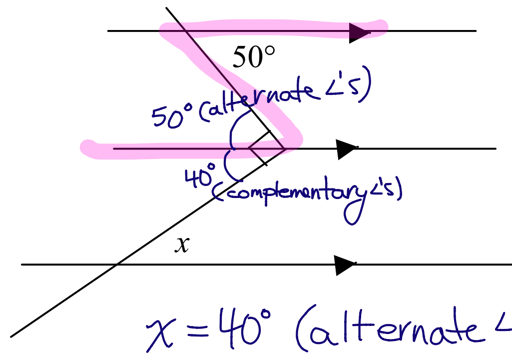


$y = 65°$ (alternate ∠'s)

$z = 55°$ (supp ∠'s)

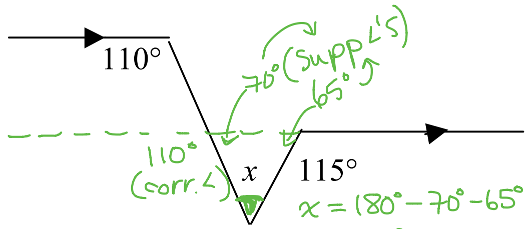
$x = 55°$ (alternate ∠'s)

d)



$x = 40°$ (alternate ∠'s)

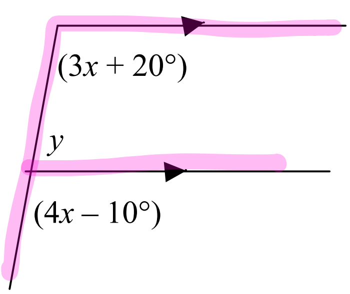
e)



$x = 180° - 70° - 65°$ (sum of int ∠'s in Δ)

$x = 45°$

f)



$3x + 20 = 4x - 10$ (corr. ∠'s)

$3x - 4x = -10 - 20$

$-x = -30$

$x = 30°$

$y + 3x + 20 = 180$ (interior ∠'s)

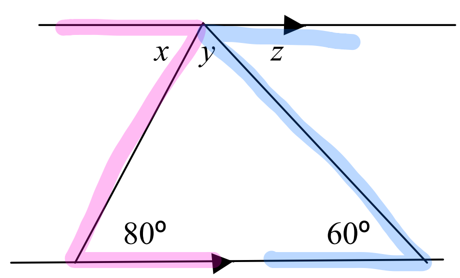
$y + 3(30) + 20 = 180$

$y + 90 + 20 = 180$

$y + 110 = 180$

$y = 70°$

g)



$x = 80°$ (alternating ∠'s)

$z = 60°$ (alternating ∠'s)

$y = 40°$ (sum of ∠'s in Δ)

The Pythagorean Theorem

The Pythagorean theorem describes both a **geometric** relationship and **numerical** relationship between the three sides of a right triangle.

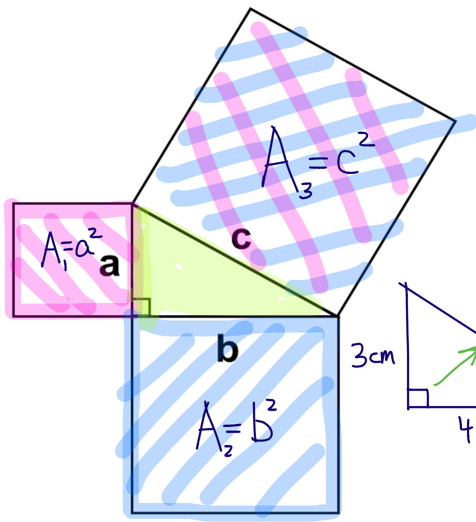
i) Geometric relationship:

The square on the hypotenuse is the sum of the squares on the other two sides.

ii) Numerical relationship:

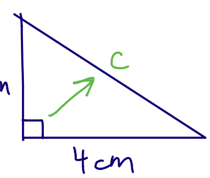
$c^2 = a^2 + b^2$, where c, is *the hypotenuse* and a and b are the *legs* of the right triangle.

· side opposite the 90° angle
· longest side



When given two sides of a right triangle, the third side can be found using the Pythagorean theorem.

Ex. 1. Find the measure of the hypotenuse, c, given a= 4 cm and b = 3 cm.



Find c:

$c^2 = a^2 + b^2$

$c^2 = (4)^2 + (3)^2$

$c^2 = 16 + 9$

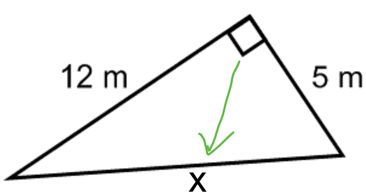
$\sqrt{c^2} = \sqrt{25}$

$\therefore c = 5$ cm

$1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$, $8^2=64$, $9^2=81$, $10^2=100$,
$11^2=121$, $12^2=144$, $13^2=169$, $14^2=196$, $15^2=225$, $16^2=256$, $17^2=289$, $18^2=324$, $19^2=361$, $20^2=400$

Ex. 2. Find x in each of the following. Round to the nearest unit if necessary.

a)



$c^2 = a^2 + b^2$

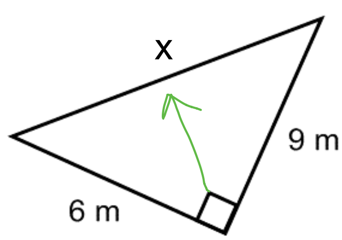
$x^2 = (5)^2 + (12)^2$

$x^2 = 25 + 144$

$\sqrt{x^2} = \sqrt{169}$

$\therefore x = 13$ m

b)



$c^2 = a^2 + b^2$

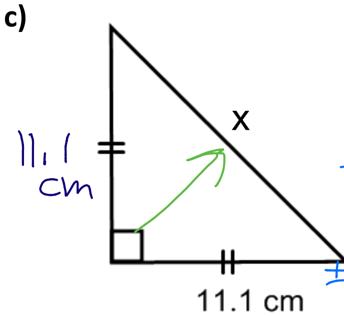
$x^2 = (9)^2 + (6)^2$

$x^2 = 81 + 36$ ← $\sqrt{100} = 10$

$\sqrt{x^2} = \sqrt{117}$ ← $\sqrt{121} = 11$

$\therefore x \doteq 11$ m

c)



11.1
× 11.1 } 2 dec places
111
1110
+ 11100
123.21

$c^2 = a^2 + b^2$

$x^2 = (11.1)^2 + (11.1)^2$

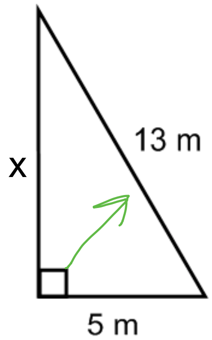
$x^2 = 123.21 + 123.21$ ← $\sqrt{225} = 15$

$\sqrt{x^2} = \sqrt{246.42}$ ← $\sqrt{256} = 16$

$\therefore x \doteq 16$ cm

Ex. 3. Find x in each of the following. Round to the nearest unit if necessary.

a)



$a^2+b^2=c^2$

$(5)^2+x^2=(13)^2$

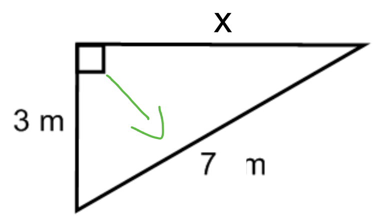
$25+x^2=169$

$x^2=169-25$

$\sqrt{x^2}=\sqrt{144}$

$\therefore x=12$ m

b)



$b^2=c^2-a^2$

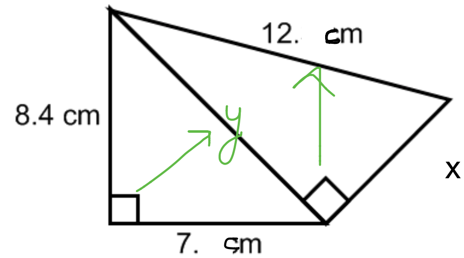
$x^2=(7)^2-(3)^2$

$x^2=49-9$

$\sqrt{x^2}=\sqrt{40}$

$\therefore x \doteq 6$ cm

c)



Find y^2:

$c^2=a^2+b^2$

$y^2=(8.4)^2+(7)^2$

$y^2=70.56+49$

$y^2=119.56$

Find x:

$b^2=c^2-a^2 \leftarrow y^2$

$x^2=(12)^2-119.56$

$x^2=144-119.56$

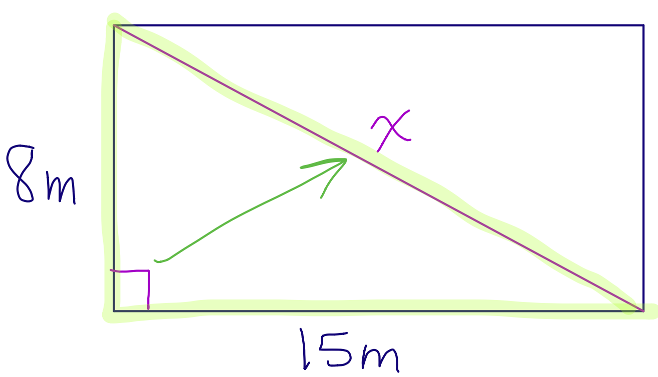
$\sqrt{x^2}=\sqrt{24.44}$

$\therefore x \doteq 5$cm

$$\begin{array}{r} 8.4 \\ \times\ 8.4 \\ \hline 336 \\ 6720 \\ \hline 70.56 \end{array}$$

$$\begin{array}{r} 144.00 \\ -119.56 \\ \hline 24.44 \end{array}$$

Ex. 4. Determine the length of the diagonal of a 15 m by 8 m rectangle.



Let x represent the length of the diagonal, in m.

$c^2=a^2+b^2$

$x^2=(15)^2+(8)^2$

$x^2=225+64$

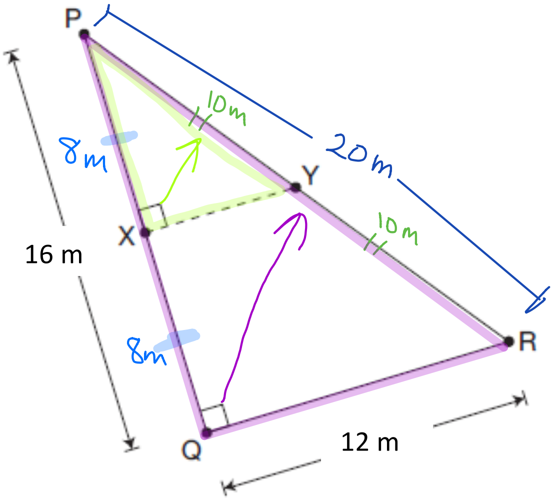
$\sqrt{x^2}=\sqrt{289}$

$x=17$

$\therefore$ the diagonal has a length of 17 m

Applications of the Pythagorean Theorem

Ex. 1. Consider the right triangle below. Line segment **XY** connects the midpoint of **PQ** to the midpoint of **PR**. What is the length of **XY**?



In $\triangle PQR$, find PR

$c^2 = a^2 + b^2$

$PR^2 = (12)^2 + (16)^2$

$PR^2 = 144 + 256$

$\sqrt{PR^2} = \sqrt{400}$

$PR = 20\,m$

In $\triangle PXY$, find XY

$a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$

$XY^2 = (10)^2 - (8)^2$

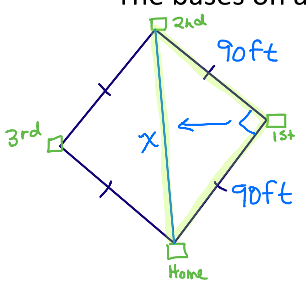
$XY^2 = 100 - 64$

$\sqrt{XY^2} = \sqrt{36}$

$\therefore XY = 6\,m$

Ex. 2. (② from Worksheet #2)

The bases on a baseball diamond are 90 feet apart. How far is it from home plate to second base?



Let x represent the distance from home plate to second base, in feet.

$c^2 = a^2 + b^2$

$x^2 = (90)^2 + (90)^2$

$x^2 = 8100 + 8100$

$\sqrt{x^2} = \sqrt{16\,200}$

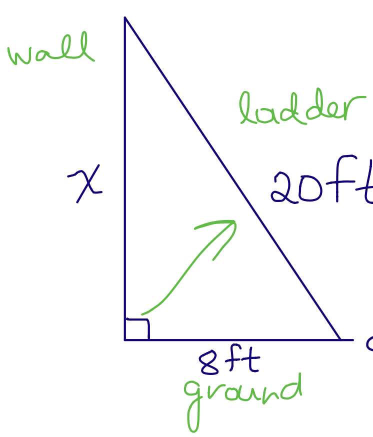
$x = \sqrt{16\,200}$

$x \doteq 127$

$\therefore$ the distance from home plate to second base is 127 feet

Ex. 3. (⑤ from Worksheet #3)

A 20-foot ladder is leaned against a wall. If the base of the ladder is 8 feet from the base of the wall, how high up on the wall will the ladder reach?



Let x represent the height of the ladder leaning on the wall, in feet.

$b^2 = c^2 - a^2$

$x^2 = (20)^2 - (8)^2$

$x^2 = 400 - 64$

$\sqrt{x^2} = \sqrt{336}$

$x = \sqrt{336}$

$x \doteq 18.3$

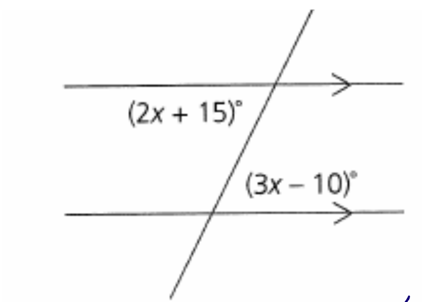
$\therefore$ the ladder will reach approximately 18.3 feet up the wall

Review: Angle Properties and The Pythagorean Theorem

Warm-up:

1. Determine the value of *x* for each of the following and state each angle property used.

a)



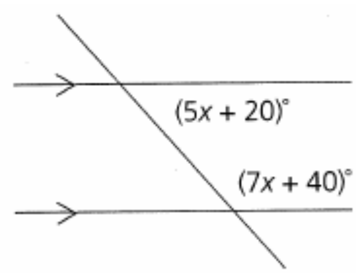
$2x+15 = 3x-10$ (alternate angles) Z-pattern

$2x-3x = -10-15$

$\frac{-x}{-1} = \frac{-25}{-1}$

$\boxed{\therefore x = 25^\circ}$

b)



$(5x+20)+(7x+40)=180$ (interior angles add to 180°) C-pattern

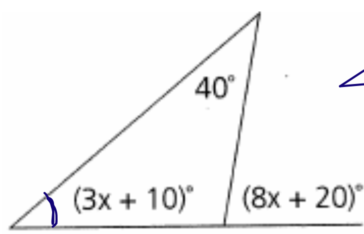
$12x+60=180$

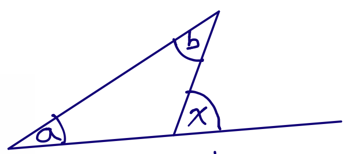
$12x=180-60$

$\frac{12x}{12}=\frac{120}{12}$

$\boxed{\therefore x=10^\circ}$

c)





$x=a+b$

$(8x+20)=(3x+10)+40$ (Exterior ∠ of a △)

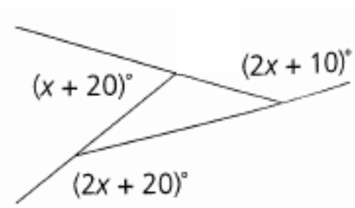
$8x+20=3x+50$

$8x-3x=50-20$

$\frac{5x}{5}=\frac{30}{5}$

$\boxed{\therefore x=6^\circ}$

d)



$(x+20)+(2x+20)+(2x+10)=360$ (sum of exterior ∠'s)

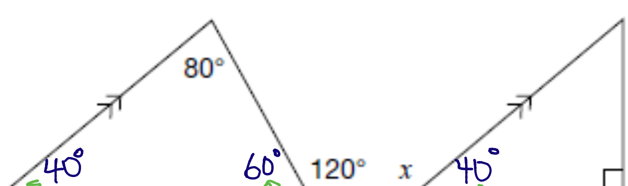
$5x+50=360$

$5x=360-50$

$\frac{5x}{5}=\frac{310}{5}$

$\boxed{\therefore x=62^\circ}$

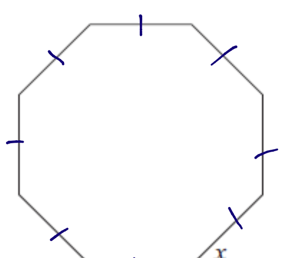
e)



② Sum of ∠'s in △ ① supplementary ∠'s ③ Corresponding angles (F-pattern)

④ $x=140^\circ$ (supplementary ∠'s)

f)

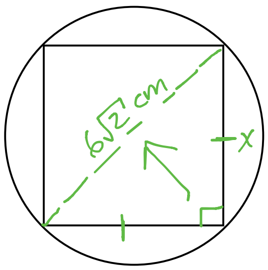


$x=\frac{360}{8}$ (sum of exterior ∠'s)

$x=\frac{90}{2}$

$\boxed{\therefore x=45^\circ}$

2. Determine the dimensions of the largest square peg that can be made from a round peg of diameter $6\sqrt{2}$ cm.

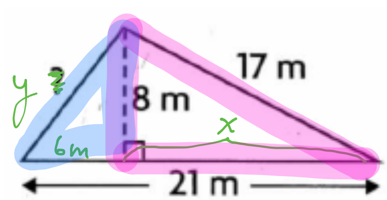


Let x represent the side length of the square, in cm

$$a^2+b^2=c^2$$
$$x^2+x^2=(6\sqrt{2})^2$$
$$2x^2=(6)^2(\sqrt{2})^2$$
$$2x^2=(36)(2)$$
$$\frac{2x^2}{2}=\frac{72}{2}$$
$$x^2=36$$
$$x=\sqrt{36}$$
$$x=6$$

$\therefore$ the largest square peg has side lengths of 6cm.

3. Determine the approximate value of the unknown side.



① Find x:

$$a^2+b^2=c^2$$
$$x^2+(8)^2=(17)^2$$
$$x^2+64=289$$
$$x^2=225$$
$$x=\sqrt{225}$$
$$\boxed{\therefore x=15\,m}$$

② Find y:

$$c^2=a^2+b^2$$
$$y^2=(6)^2+(8)^2$$
$$y^2=36+64$$
$$y^2=100$$
$$y=\sqrt{100}$$
$$\boxed{\therefore y=10\,m}$$