

## WORKSHOP: Performing Calculations Using Formulas in Geometry

### A. Substitution Rules

1. Write the formula out in its general form before substituting.
2. Place brackets around each numerical quantity/value that replaces a variable in a formula.
3. Promote any existing brackets in a formula to square brackets during your substitution step.
4. When the formula includes  $\pi$ , use the  $\pi$  button on your calculator and do **not** replace the  $\pi$  symbol with a numerical value during your substitution step (note: if your calculator does not have a  $\pi$  button, then you will replace the  $\pi$  symbol with 3.14159 and replace the = with  $\doteq$  at this step in addition to the final step).

Ex. 1. Substitute the appropriate values into each of the following formulas:

a) Given:  $C = 2\pi r$ , where  $r = 57$  cm

$$C = 2\pi r$$

$$C = 2\pi(57) \leftarrow C = 114\pi \text{ (exact value)}$$

$$C \doteq 358 \text{ cm}$$

( $C = 358.141562509\dots$ )

b) Given:  $A = \frac{(a+b)h}{2}$ , where  $a = 3.4$  mm,  $b = 2.2$  mm,  $h = 5.0$  mm

$$A = \frac{(a+b)h}{2}$$

$$A = \frac{[(3.4) + (2.2)](5.0)}{2}$$

$$A = 14 \text{ mm}^2$$

c) Given:  $V = \frac{\pi r^2 h}{3}$ , where  $r = 3.75$  m,  $h = 6.1$  m

$$V = \frac{\pi r^2 h}{3}$$

$$V = \frac{\pi(3.75)^2(6.1)}{3}$$

$$V \doteq 89.8 \text{ m}^3$$

( $V = 89.829914938\dots$ )

d) Given:  $A = \pi r^2 + 2\pi r h + \pi r s$ , where  $r = 5$  m,  $s = 8$  m,  $h = 11$  m

$$A = \pi r^2 + 2\pi r h + \pi r s$$

$$A = \pi(5)^2 + 2\pi(5)(11) + \pi(5)(8)$$

$$\left\{ \begin{array}{l} A = 25\pi + 110\pi + 40\pi \\ A = 175\pi \end{array} \right\} \text{ exact value}$$

$$A \doteq 550 \text{ m}^2$$

( $A = 549.77871437\dots$ )

## B. Rounding and Place Value Review

PLACE VALUE CHART													
Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones / Unit	Decimal Point	Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths

- When rounding to a given place value, the number directly **to the right** determines how you will round.
- If the number to the right is 5 or greater, round the required digit up one and remove all values thereafter.
  - If the number to the right is less than 5, keep the required digit the same and remove all values thereafter.

Ex. 2. Round 725.6499 to the given place value.

- a) Unit: 726                      b) Tenth: 725.6                      c) Hundredth: 725.65
- d) Thousandth: 725.650                      e) Ten: 730                      f) Whole number: 726

## C. Calculation Rules

- Before you begin your calculations, be aware of the required rounding for your final answer.
- If the problem is multi-step and requires an intermediate value (i.e. using Pythagorean theorem), round this value to at least one decimal place greater than the rounding required for your final answer.
- Perform the final calculation (i.e. perimeter, area, volume) by typing the entire expression into your calculator in one step. Note: For formulas in fraction form, you will need to type brackets around your numerator and denominator.

Ex. 3. a) Go to Ex. 1. and complete the calculations for each equation. Round questions a) and d) to the nearest unit and round questions b) and c) to the nearest tenth.

b) Given the following:  $h^2 = c^2 - b^2$ ,  $A = \frac{bh}{2}$ ,  $c = 4.68$  m, and  $b = 2.3$  m

Calculate the value of  $h$  and use it to find the value of  $A$ , to one decimal place.

2 dec. places

$$h^2 = c^2 - b^2$$

$$h^2 = (4.68)^2 - (2.3)^2$$

$$h = \sqrt{(4.68^2 - 2.3^2)}$$

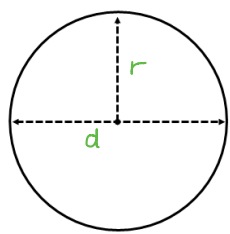
$$h = 4.08 \text{ m}$$

$$A = \frac{bh}{2}$$

$$A = \frac{(2.3)(4.08)}{2}$$

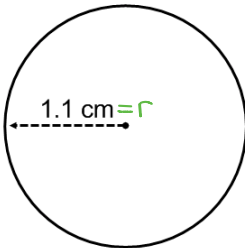
$$A = 4.7 \text{ m}^2$$

### Perimeter

<p>Formulas</p>		$C = 2\pi r \text{ or } C = \pi d$ $d = 2r$ $r = d \div 2$
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Ex. 1. Find the circumference to one decimal place.

a)

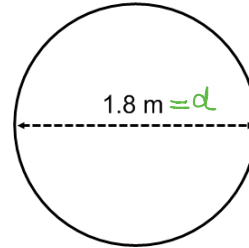


$$C = 2\pi r$$

$$C = 2\pi(1.1)$$

$$C \approx 6.9 \text{ cm}$$

b)



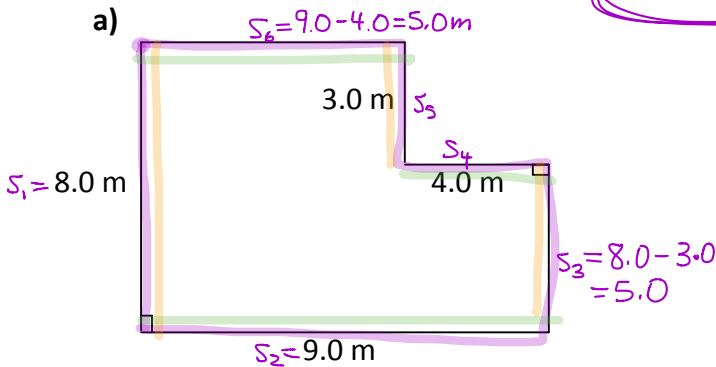
$$C = \pi d$$

$$C = \pi(1.8)$$

$$C \approx 5.7 \text{ m}$$

Ex. 2. Find the perimeter (the distance around the outside) of each figure, to one decimal place.

a)

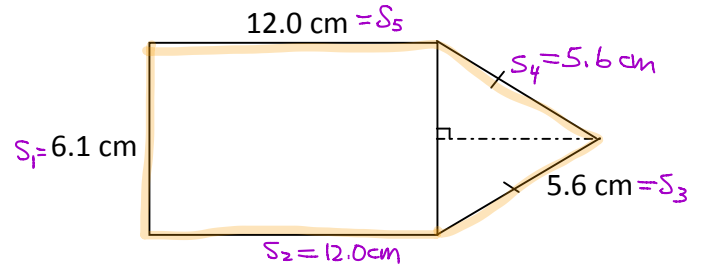


$$P = s_1 + s_2 + s_3 + s_4 + s_5 + s_6$$

$$P = 8 + 9 + 5 + 4 + 3 + 5$$

$$P = 34.0 \text{ m}$$

b)



$$P = s_1 + s_2 + s_3 + s_4 + s_5$$

$$P = 6.1 + 12.0 + 5.6 + 5.6 + 12.0$$

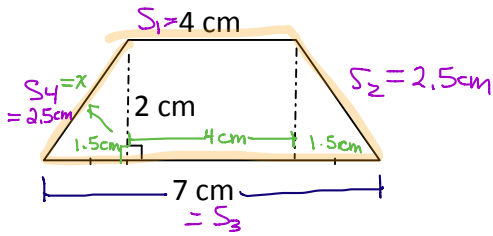
or

$$P = 6.1 + 2(12.0) + 2(5.6)$$

$$P = 41.3 \text{ cm}$$

Ex. 3. Find the perimeter of each figure, to the nearest unit.

a)

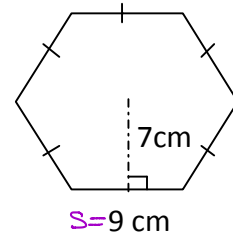


$$P = S_1 + S_2 + S_3 + S_4$$

$$P = 4 + 2.5 + 7 + 2.5$$

$$P = 16 \text{ cm}$$

b)



$$P = 6s$$

$$P = 6(9)$$

$$P = 54 \text{ cm}$$

①

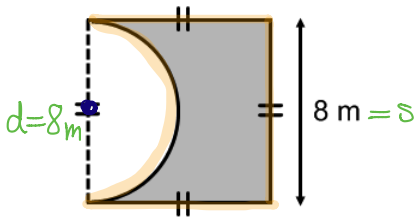
$$c^2 = a^2 + b^2$$

$$x^2 = (1.5)^2 + (2)^2$$

$$x = \sqrt{(1.5)^2 + (2)^2}$$

$$x = 2.5 \text{ cm}$$

Ex. 4. Find the perimeter of the shaded figure, to the nearest tenth.



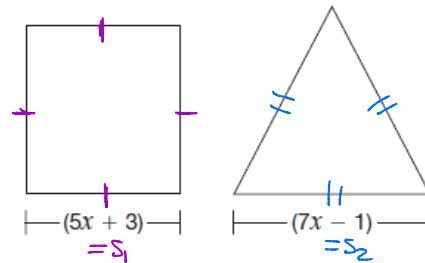
$$(P = C_{\frac{1}{2}\text{circle}} + 3s)$$

$$P = \pi d \div 2 + 3s$$

$$P = \pi(8) \div 2 + 3(8)$$

$$P \approx 36.6 \text{ m}$$

Ex. 5. A square and an equilateral triangle below have the same perimeter. Find  $x$ .



$$P_{\text{square}} = P_{\text{triangle}}$$

$$4s_1 = 3s_2$$

$$4(5x + 3) = 3(7x - 1)$$

$$20x + 12 = 21x - 3$$

$$20x - 21x = -3 - 12$$

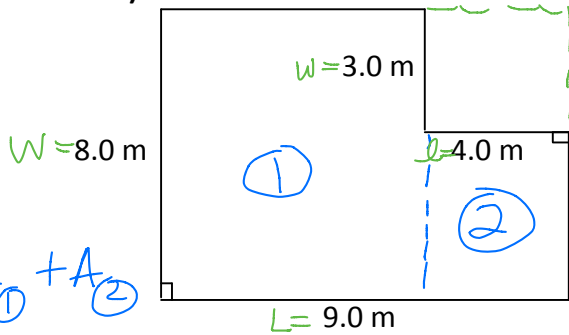
$$\frac{-x}{-1} = \frac{-15}{-1}$$

$$x = 15 \text{ units}$$

### Area

Ex. 1. Find the area of each figure to one decimal place.

a)



$$A = A_1 + A_2$$

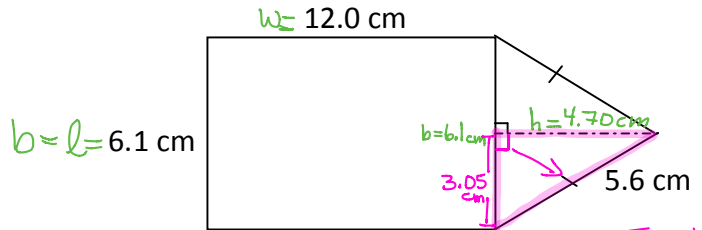
$$A_{\text{Total}} = A_{\text{large}} - A_{\text{small}}$$

$$A = LW - lw$$

$$A = (9)(8) - (4)(3)$$

$$A = 60.0 \text{ m}^2$$

b)



$$A_{\text{Total}} = A_{\square} + A_{\triangle}$$

$$A = lw + \frac{bh}{2}$$

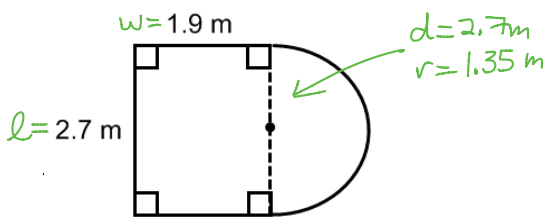
$$A = (12)(6.1) + \frac{(6.1)(4.70)}{2}$$

$$A \approx 87.5 \text{ cm}^2$$

Find h:  
 $a^2 = c^2 - b^2$   
 $h^2 = (5.6)^2 - (3.05)^2$   
 $h = \sqrt{(5.6)^2 - (3.05)^2}$   
 $h \approx 4.70$

Ex. 2. Find the area of each figure to the nearest unit.

a)



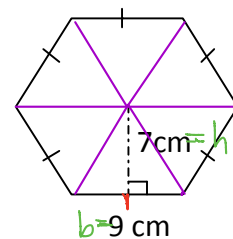
$$A_{\text{total}} = A_{\square} + A_{\frac{1}{2}\circ}$$

$$A = lw + \frac{\pi r^2}{2}$$

$$A = [2.7)(1.9)] + \left[ \frac{\pi(1.35)^2}{2} \right]$$

$$A \approx 8 \text{ m}^2$$

b)



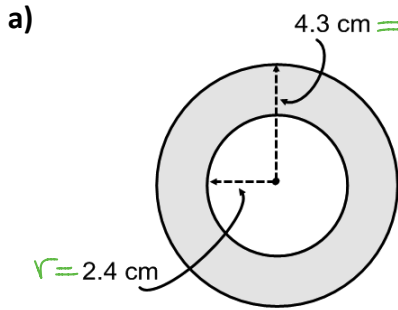
$$A_{\text{total}} = A_{6\triangle}$$

$$A = 6 \left( \frac{bh}{2} \right)$$

$$A = \frac{6}{1} \left[ \frac{(9)(7)}{2} \right]$$

$$A = 189 \text{ cm}^2$$

Ex. 3. Find the area of the shaded region to one decimal place.

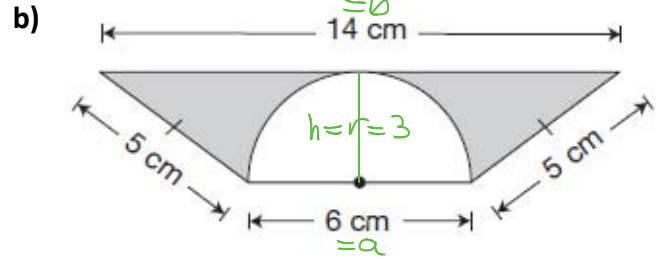


$$A_{\text{total}} = A_{\text{large circle}} - A_{\text{small circle}}$$

$$A = \pi R^2 - \pi r^2$$

$$A = \pi (4.3)^2 - \pi (2.4)^2$$

$$A \approx 40.0 \text{ cm}^2$$



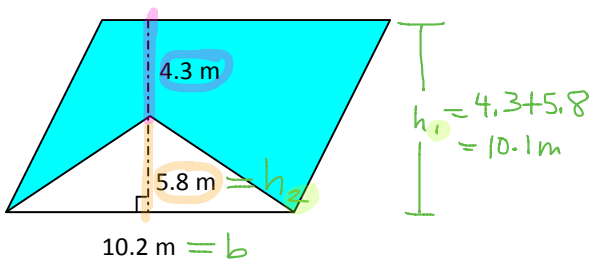
$$A_{\text{total}} = A_{\text{trapezoid}} - A_{\frac{1}{2}O}$$

$$A = \frac{h(a+b)}{2} - \frac{\pi r^2}{2}$$

$$A = \frac{3(6+14)}{2} - \frac{\pi(3)^2}{2}$$

$$A \approx 15.9 \text{ cm}^2$$

Ex. 4. Find the shaded area to one decimal place.



$$A = A_{\text{parallelogram}} - A_{\Delta}$$

$$A = bh_1 - \frac{bh_2}{2}$$

$$A = (10.2)(10.1) - \frac{(10.2)(5.8)}{2}$$

$$A \approx 73.4 \text{ m}^2$$

Ex. 5. Find the radius and diameter of a circle with an area of  $78.5 \text{ m}^2$  to one decimal place.

$$A = \pi r^2$$

If  $A = 78.5 \text{ m}^2$  find  $r$

$$\frac{78.5}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{78.5}{\pi}} = \sqrt{r^2}$$

$$r = \sqrt{\frac{78.5}{\pi}}$$

$$r \approx 5.0 \text{ m}$$

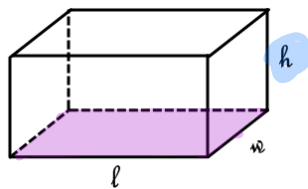
$$\therefore d \approx 10.0 \text{ m}$$

# Volume

The volume of a prism or cylinder is the product of the area of the base and the height.

## Prisms

A.

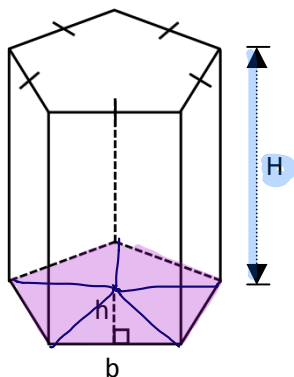


$$V = A_{\text{base}} \times \text{Height}$$

$$V = lw \times h$$

$$\therefore V = lwh$$

B.



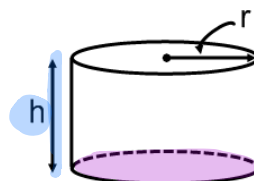
$$V = A_{\text{base}} \times \text{Height}$$

$$V = (5A_{\Delta}) \times \text{Height}$$

$$V = 5\left(\frac{bh}{2}\right) \times H$$

$$V = \frac{5bhH}{2}$$

## Cylinders



$$V = A_{\text{base}} \times \text{Height}$$

$$V = \pi r^2 \times h$$

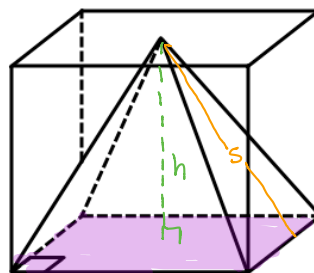
$$V = \pi r^2 h$$

## Pyramids

All pyramids have triangular sides.  
The base can be any polygon.

The volume of a pyramid is  $\frac{1}{3}$  the volume of a prism with the same base and height.

$$V_{\text{pyramid}} = \frac{1}{3} A_{\text{base}} \times \text{height}$$

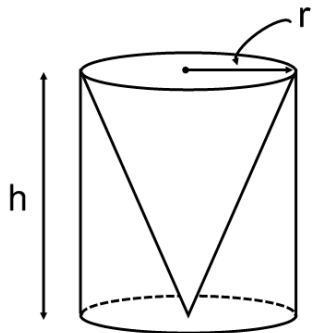


\* may need to calculate the height using Pythagorean Theorem

## Cones

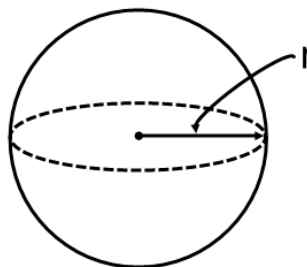
The volume of a cone is  $\frac{1}{3}$  the volume of a cylinder with the same radius and height.

$$V_{\text{cone}} = \frac{1}{3} A_{\text{base}} \times \text{height} \text{ or } V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$



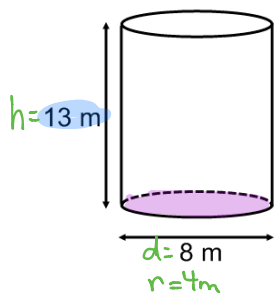
## Spheres

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$



Ex. 1. Calculate each volume, to one decimal place.

a)



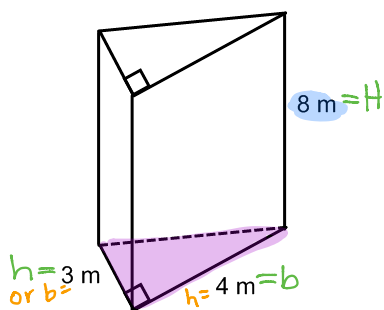
$$V = A_{\text{base}} \times H$$

$$V = \pi r^2 h$$

$$V = \pi (4)^2 (13)$$

$$\therefore V \doteq 653.5 \text{ m}^3$$

b)



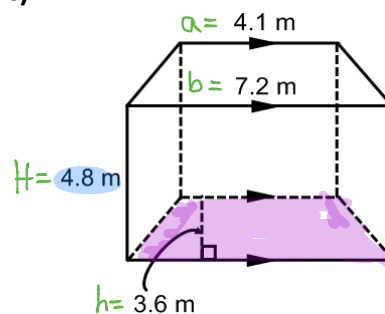
$$V = A_{\text{base}} \times H$$

$$V = \frac{bh}{2} \times H$$

$$V = \frac{3 \times 4 \times 8}{2}$$

$$\therefore V = 48 \text{ m}^3$$

c)



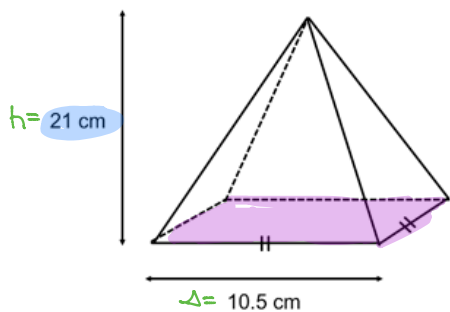
$$V = A_{\text{base}} \times H$$

$$V = \frac{(a+b)h}{2} \times H$$

$$V = \frac{(4.1 + 7.2)(3.6)(4.8)}{2}$$

$$\therefore V \doteq 97.6 \text{ m}^3$$

d)



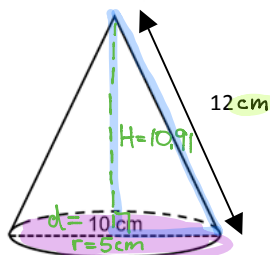
$$V = \frac{1}{3} A_{\text{base}} \times H$$

$$V = \frac{1}{3} s^2 \times h$$

$$V = \frac{(10.5)^2 (21)}{3}$$

$$\therefore V \doteq 771.8 \text{ cm}^3$$

e)



Find h:

$$a^2 + b^2 = c^2$$

$$5^2 + h^2 = 12^2$$

$$h^2 = 144 - 25$$

$$h^2 = 119$$

$$h = \sqrt{119}$$

$$h = 10.91$$

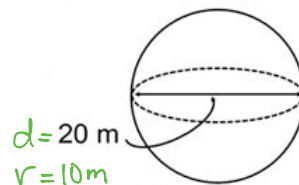
$$V = \frac{1}{3} A_{\text{base}} \times H$$

$$V = \frac{\pi r^2 h}{3}$$

$$V = \frac{\pi (5)^2 (10.91)}{3}$$

$$\therefore V \doteq 285.6 \text{ cm}^3$$

f)



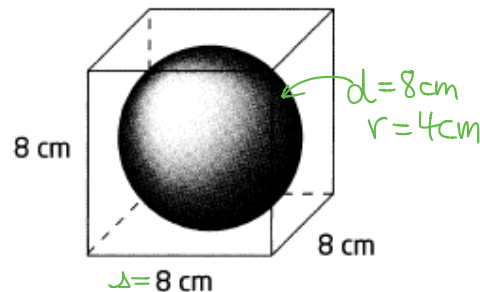
$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4\pi (10)^3}{3}$$

$$\therefore V \doteq 4188.8 \text{ m}^3$$



Ex. 2. A sphere just fits inside an 8 cm by 8 cm by 8 cm cubic box.  
What percent of the box is empty to the nearest percent?



$$\begin{aligned} \textcircled{1} V_{\text{empty}} &= V_{\text{box}} - V_{\text{sphere}} \\ &= \Delta^3 - \frac{4\pi r^3}{3} \\ &= (8)^3 - \frac{4\pi(4)^3}{3} \end{aligned}$$

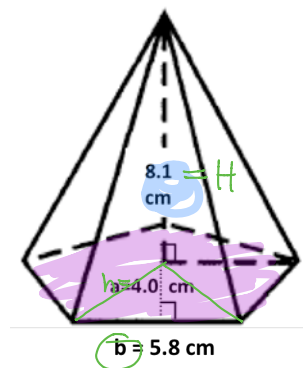
$$\begin{aligned} \textcircled{2} \% \text{ Empty} &= \frac{V_{\text{empty}}}{V_{\text{box}}} \times 100\% \\ &= \frac{243.9}{(8)^3} \times 100\% \end{aligned}$$

$$V_{\text{empty}} \doteq 243.9 \text{ cm}^3$$

$$\doteq 48\%$$

$\therefore$  approx 48% of the box is empty.

Ex. 3. An oil lamp with a reservoir in the shape of a pyramid has a regular pentagonal base as shown and a height of 8.1 cm. If the oil comes in 750 ml bottles, how many times can the lamp be completely filled with one bottle of oil? (1 ml = 1 cm<sup>3</sup>)



$$\begin{aligned} \textcircled{1} V_{\text{reservoir}} &= \frac{1}{3} A_{\text{base}} \times \text{Height} \\ &= \frac{1}{3} \left[ \frac{5bh}{2} \right] \times H \end{aligned}$$

$$V = \frac{1}{3} \left[ \frac{5(4.0)(5.8)}{2} \right] (8.1)$$

$$V = \frac{5(4.0)(5.8)(8.1)}{(3)(2)}$$

$$V \doteq 156.6 \text{ cm}^3$$

$$\begin{aligned} \textcircled{2} V_{\text{bottle}} &= 750 \text{ mL} \\ &= 750 \text{ cm}^3 \end{aligned}$$

$$\textcircled{3} \# \text{ of times filled} = \frac{750}{156.6}$$

$$\doteq 4.8$$

$\therefore$  the lamp can be filled completely

4 times.

Ex. 4. The volume of a rectangular prism is represented by  $12x^3$ . Determine an expression for the area of the base if the height is represented by  $3x$ .

$$V = lwh$$

$$V = A_{\text{base}} \times \text{Height}$$

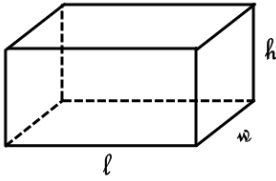
$$\frac{12x^3}{3x} = \frac{A(3x)}{3x}$$

$$A = 4x^2$$

$\therefore$  the area of the base is  $4x^2$  units<sup>2</sup>.

# Surface Area

## Rectangular Prism



S.A. =  $A_{\text{left \& right}} + A_{\text{bottom \& top}} + A_{\text{front \& back}}$

$$S.A. = 2wh + 2lw + 2lh$$

or

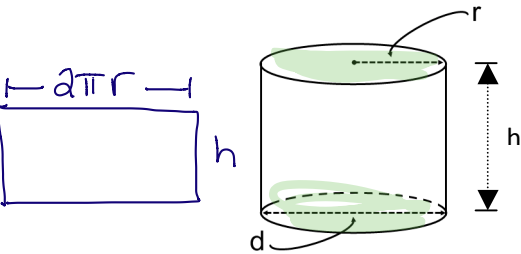
$$SA = 2(wh + lw + lh)$$

## Cylinder

S.A. =  $A_{\text{top}} + A_{\text{base}} + A_{\text{lateral surface}}$

$$SA = \pi r^2 + \pi r^2 + 2\pi r h$$

$$SA = 2\pi r^2 + 2\pi r h$$

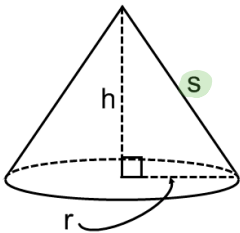


## Cone

S.A. =  $A_{\text{base}} + A_{\text{lateral surface}}$

$$SA = \pi r^2 + \pi r s$$

\* may need to use Pyth. Thm. to calculate s if given h.

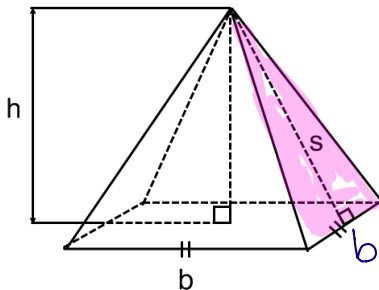


## Pyramid (Square based)

S.A. =  $A_{\text{bottom}} + 4A_{\text{side}}$

$$SA = b^2 + 4 \left( \frac{bs}{2} \right)$$

$$SA = b^2 + 2bs$$

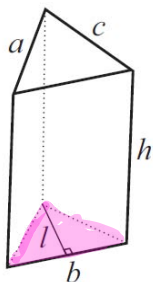


## Triangular Prism

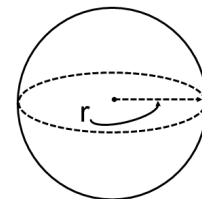
S.A. =  $A_{\text{top}} + A_{\text{bottom}} + (A_{\text{rectangles}})$

$$SA = \frac{bl}{2} + \frac{bl}{2} + (bh + ch + ah)$$

$$SA = bl + bh + ch + ah$$

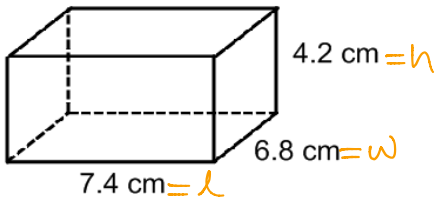


Sphere S.A. =  $4\pi r^2$



Ex. 1. Calculate the total surface area to one decimal place.

a)

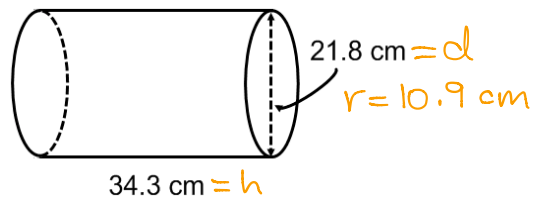


$$SA = 2wh + 2lw + 2lh$$

$$SA = 2(6.8)(4.2) + 2(7.4)(6.8) + 2(7.4)(4.2)$$

$$SA \doteq 219.9 \text{ cm}^2$$

b)

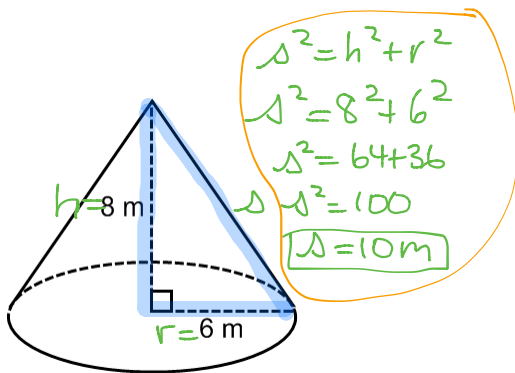


$$SA = 2\pi r^2 + 2\pi rh$$

$$SA = 2\pi(10.9)^2 + 2\pi(10.9)(34.3)$$

$$SA \doteq 3095.6 \text{ cm}^2$$

c)



$$s^2 = h^2 + r^2$$

$$s^2 = 8^2 + 6^2$$

$$s^2 = 64 + 36$$

$$s^2 = 100$$

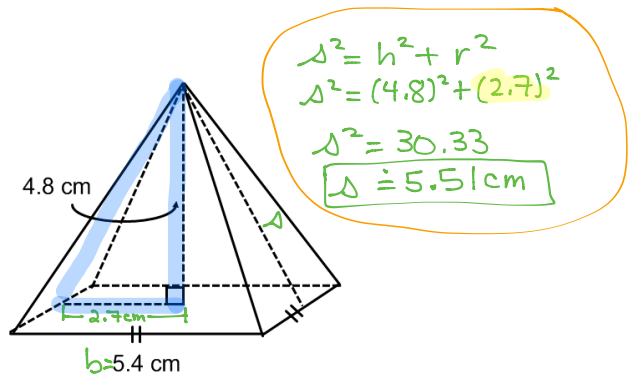
$$s = 10 \text{ m}$$

$$SA = \pi r^2 + \pi rs$$

$$SA = \pi(6)^2 + \pi(6)(10)$$

$$SA \doteq 301.6 \text{ m}^2$$

d)



$$s^2 = h^2 + r^2$$

$$s^2 = (4.8)^2 + (2.7)^2$$

$$s^2 = 30.33$$

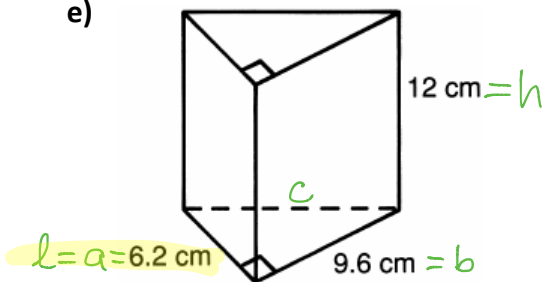
$$s \doteq 5.51 \text{ cm}$$

$$SA = b^2 + 2bs$$

$$SA = (5.4)^2 + 2(5.4)(5.51)$$

$$SA \doteq 88.7 \text{ cm}^2$$

e)



$$c^2 = a^2 + b^2$$

$$c^2 = (6.2)^2 + (9.6)^2$$

$$c^2 = 130.6$$

$$c \doteq 11.43 \text{ cm}$$

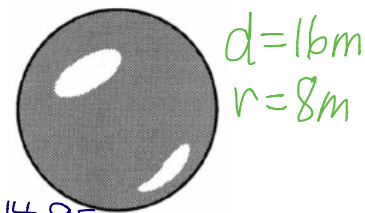
$$SA = bl + ah + bh + ch$$

$$SA = (9.6)(6.2) + (6.2)(12) + (9.6)(12) + (11.43)(12)$$

$$SA \doteq 386.3 \text{ cm}^2$$

Ex. 2. A spherical weather balloon 16 m in diameter is to be painted with a special reflective coating.

- Determine the area, to the nearest square metre, to be painted.
- If 1 tin of paint covers  $20 \text{ m}^2$ , how many tins of paint are required?
- What is the total cost of painting the balloon if each tin costs \$14.95?



$$\begin{aligned} \text{a) } SA &= 4\pi r^2 \\ SA &= 4\pi(8)^2 \end{aligned}$$

$$SA \doteq 804$$

$\therefore$  approx  $804 \text{ m}^2$  of surface area needs to be painted

$$\begin{aligned} \text{b) } \# \text{ tins} &= \frac{804}{20} \\ &\doteq 40.2 \end{aligned}$$

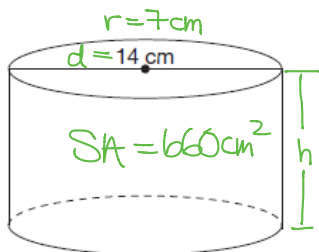
$\therefore$  41 tins of paint are needed.

$$\begin{aligned} \text{c) } \text{Cost} &= 41 \times 14.95 \\ &= 612.95 \end{aligned}$$

$\therefore$  the total cost is \$612.95

with tax:  $612.95 \times 1.13$

Ex. 3. The cylinder pictured below has a surface area of  $660 \text{ cm}^2$ . Determine the height of the cylinder to one decimal place.



$$SA = 2\pi r^2 + 2\pi r h$$

$$660 = 2\pi(7)^2 + 2\pi(7)h$$

$$660 = 98\pi + 14\pi h$$

$$\frac{660 - 98\pi}{14\pi} = \frac{14\pi h}{14\pi}$$

$$\frac{660 - 98\pi}{14\pi} = h$$

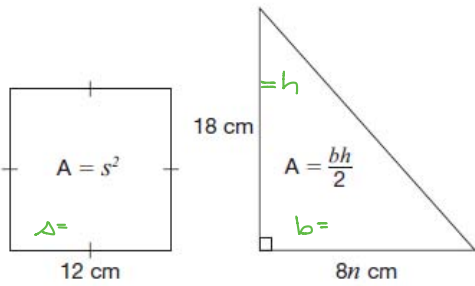
$$h \doteq \underline{\underline{8.0 \text{ cm}}}$$

Ex. 4. If the radius of a sphere is doubled, the surface area will increase by a factor of 4 and the volume will increase by a factor of 8.

$$V = \frac{4}{3}\pi r^3$$

### Applications: Perimeter, Area, Surface Area and Volume

Ex. 1. A square and the triangle below have the same area. Find  $n$ .



$$A_{\text{square}} = A_{\text{triangle}}$$

$$s^2 = \frac{bh}{2}$$

$$(12)^2 = \frac{(18)(8n)}{2}$$

$$\frac{144}{72} = \frac{72n}{72}$$

$$\therefore n = 2$$

Ex. 2. The distance covered in 5 laps of a circular track is  $400\pi$  metres.

What is the shortest distance between any point on the track and the centre to the nearest tenth of a metre?

radius

$$\begin{aligned} \textcircled{1} 1 \text{ lap} &= C \\ &= 400\pi \div 5 \end{aligned}$$

$$\therefore 1 \text{ lap} = 80\pi$$

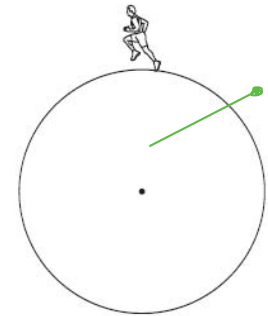
$$\therefore C = 80\pi$$

$$\begin{aligned} \textcircled{2} C &= 2\pi r \\ \text{Find } r \text{ if } \\ C &= 80\pi \end{aligned}$$

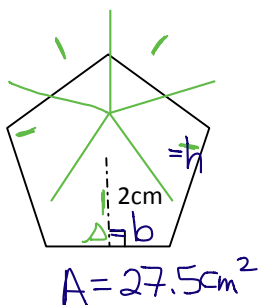
$$\frac{80\pi}{2\pi} = \frac{2\pi r}{2\pi}$$

$$40 = r$$

$\therefore$  the shortest distance from the centre is 40m.



Ex. 3. Find the side length of the given regular polygon if the area is  $27.5 \text{ cm}^2$  (to one decimal place).



$$A = 27.5 \text{ cm}^2$$

$$A_{\text{pentagon}} = 5 \times A_{\text{triangles}}$$

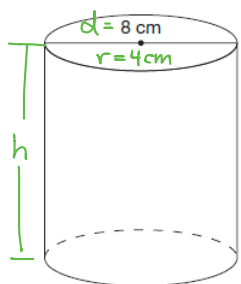
$$A = \left(\frac{bh}{2}\right) 5$$

$$27.5 = \frac{b(2)}{2} (5)$$

$$\frac{27.5}{5} = \frac{5b}{5}$$

$$\therefore b = 5.5 \text{ cm}$$

Ex. 4. The cylinder below has a volume of  $150 \text{ cm}^3$ . Find the lateral surface area to the nearest hundredth.



② Lateral SA:  
 $SA = 2\pi r h$   
 $= 2\pi(4)(2.984)$   
 $SA \doteq 75.00 \text{ cm}^2$

①  $V = A_{\text{base}} \times H$

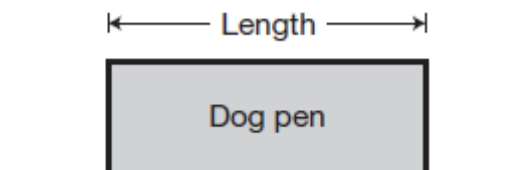
$V = \pi r^2 h$   
 Find  $h$ , if  $r = 4 \text{ cm}$ ,  $V = 150 \text{ cm}^3$

$150 = \frac{\pi(16)h}{16\pi}$

$\frac{150}{16\pi} = h$

$h \doteq 2.984 \text{ cm}$

Ex. 6. Marcus is building a rectangular dog pen along the side of his house as shown below.



House  
 For 3 sides; max area:  
 $l = 2w$

Marcus has 20 m of fencing for the 3 sides of the dog pen.

What is the length of the dog pen with the maximum area?

~~a~~ 4 m  $A = l \times w$   
 $4 \times 8 = 32$

~~b~~ 5 m  $5 \times 7.5 = 37.5$

**c** 10 m  $10 \times 5 = 50$

~~d~~ 12 m  $12 \times 4 = 48$

Find  $w$ :

$l + 2w = 20$

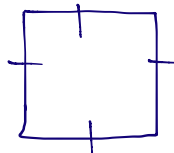
$\frac{2w}{2} = \frac{20-l}{2}$

$w = \frac{20-l}{2}$

Ex. 7. a) Determine the maximum area of a rectangle with a perimeter of 32 cm.

Note: The rectangle with a maximum area for a given perimeter is a square.

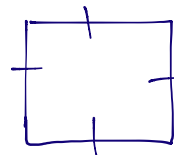
$P = 32$   $S = \frac{P}{4}$   $A = s^2$   
 $S = \frac{32}{4}$   $S = 8 \text{ cm}$   $= (8)^2$   
 $S = 8 \text{ cm}$   $= 64 \text{ cm}^2$   
 $\therefore$  the max. area is  $64 \text{ cm}^2$  if  $P = 32 \text{ cm}$ .



b) Determine the minimum length of wood needed to build a rectangular frame for an area of  $132 \text{ cm}^2$ .

Note: The rectangle with a minimum perimeter for a given area is a square.

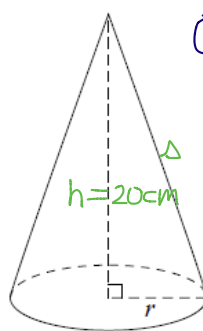
$A = 132 \text{ cm}^2$   $s^2 = 132$   
 $s = \sqrt{132}$   
 $s \doteq 11.5 \text{ cm}$



$P = 4(11.5)$   
 $P = 46$

$\therefore$  the minimum length of wood needed is 46 cm.

Ex. 5. The cone shown is 20 cm high and has a total volume of  $1000 \text{ cm}^3$ . Determine the slant height to the nearest tenth.



①  $V = \frac{\pi r^2 h}{3}$   
 Find  $r$  if  $V = 1000 \text{ cm}^3$ ,  $h = 20 \text{ cm}$

$3(1000) = \frac{\pi r^2 (20)}{3}$

$\frac{3000}{20\pi} = \frac{20\pi r^2}{20\pi}$

$\sqrt{\frac{3000}{20\pi}} = \sqrt{r^2}$

$r \doteq 6.91 \text{ cm}$

② Find  $s$ :

$s^2 = h^2 + r^2$

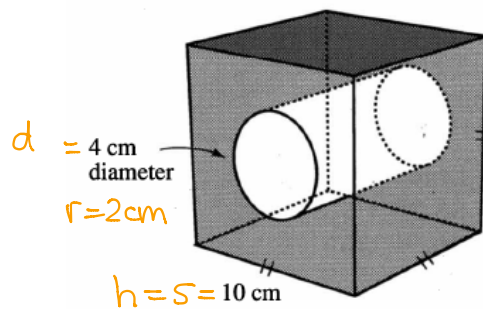
$s = \sqrt{(20)^2 + (6.91)^2}$

$s \doteq 21.2 \text{ cm}$

**Ex. 8.** A metal casting has dimensions as shown. If the density of the material is  $6.8 \text{ g/cm}^3$ , determine the mass of the casting, in kg.

**Note:** Mass = Density x Volume

$$\begin{aligned} \textcircled{1} \quad V_{\text{casting}} &= V_{\text{cube}} - V_{\text{cylinder}} \\ &= s^3 - \pi r^2 h \\ &= (10)^3 - \pi(2)^2(10) \\ &= 1000 - (40\pi) \\ \therefore V &= 874.336 \text{ cm}^3 \end{aligned}$$

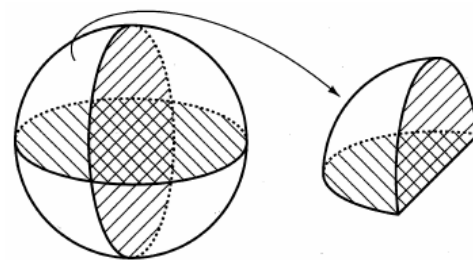


$$\begin{aligned} \textcircled{2} \quad \text{Mass} &= \text{Density} \times \text{Volume} \\ M &= 6.8 \text{ g/cm}^3 \times 874.336 \text{ cm}^3 \\ \therefore M &= 5945.4848 \text{ g} \\ M_{\text{mass}} &= \frac{5945.4848}{1000} \\ &= 5.945 \end{aligned}$$

$\therefore$  the mass of the casting is approx 5.9 kg.

**Ex. 9.** A window display is made from a Styrofoam ball 1.8 m in diameter. The ball is cut into 4 equal parts as shown.

Determine the cost of covering the one quarter sphere on all 3 surfaces with "glitter" paint, if one tin of paint covers  $1.5 \text{ m}^2$ , and costs \$4.95. Include 13% tax in the total cost.



$$\begin{aligned} \textcircled{1} \quad SA &= SA_{\frac{1}{4}\text{sphere}} + 2A_{\frac{1}{2}\text{circle}} \\ &= \frac{4\pi r^2}{4} + \frac{2\pi r^2}{2} \\ &= 2\pi r^2 \end{aligned}$$

Find SA if  $r = 0.9 \text{ m}$

$$SA = 2\pi(0.9)^2$$

$$\therefore SA = 5.09 \text{ m}^2$$

$$\begin{aligned} \textcircled{2} \quad \# \text{ tins} &= SA \div 1.5 \\ &= 5.09 \div 1.5 \\ &= 3.4 \end{aligned}$$

$\therefore$  4 full tins will be needed.

$$\begin{aligned} \textcircled{3} \quad \text{Total Cost} &= 4 \times 4.95 \times 1.13 \\ &= 22.374 \end{aligned}$$

$\therefore$  The total cost with tax is \$22.37.