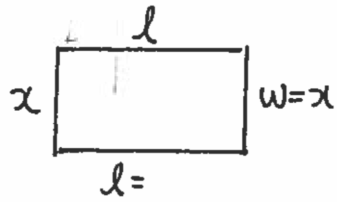
Ex. 5.5 Solutions pg. 206

#1.



Find l in terms of x

$P = 100$ cm

$2l + 2x = 100$

$l = 50 - x$

Maximize the area, A in cm^2

$A = lw$

$A = (50-x)(x)$

$A = -x^2 + 50x$

$\frac{dA}{dx} = \ ?$

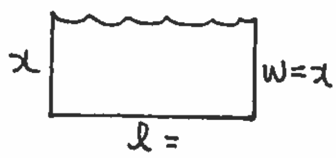
For max area

$\frac{dA}{dx} = 0$

Note: $0 \le x \le 50$

x	A
0	
?	
50	

#3.



Find l in terms of x

600m of fencing

$l + 2x = 600$

$l = 600 - 2x$

Maximize the area, A in m^2

$A = lw$

$A = (600-2x)(x)$

$A = -2x^2 + 600x$

$\frac{dA}{dx} = \ ?$

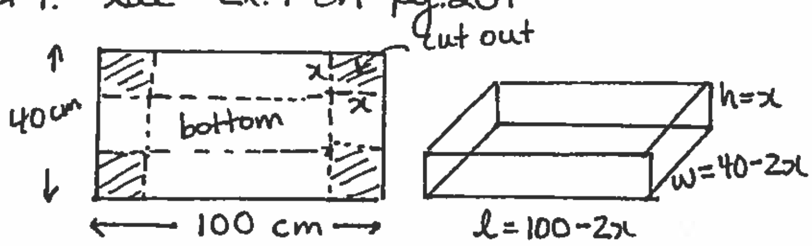
For max. area

$\frac{dA}{dx} = 0$

Note: $0 \le x \le 300$

x	A
0	
?	
300	

#4. see Ex. 1 on pg. 204



Maximize the volume V in cm^3

$V = lwh$

$V = (100-2x)(40-2x)(x)$

$V = 4x^3 - 280x^2 + 4000x$

$\frac{dV}{dx} = \ ?$

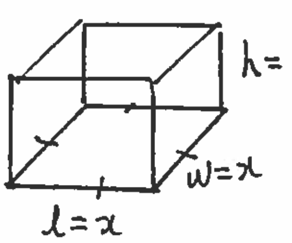
For max volume

$\frac{dV}{dx} = 0$

Note: $0 \le x \le 20$

x	V
0	
?	
20	

#5.



Find h in terms of x

$V = 1000\ cm^3$

$x \cdot x \cdot h = 1000$

$h = \frac{1000}{x^2}$

Minimize the surface area S.A in cm^2

$S.A = 2x^2 + 4xh$

$S.A = 2x^2 + 4x\left(\frac{1000}{x^2}\right)$

$S.A = 2x^2 + 4000x^{-1}$

$\frac{dS.A}{dx} = \ ?$

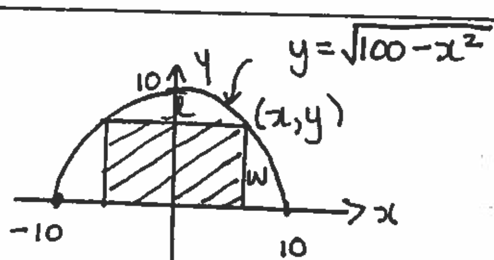
For min. S.A

$\frac{dS.A}{dx} = 0$

Note: $2 \le x \le \sqrt{1000}$

x	V
2	
?	
$\sqrt{1000}$	

#6.



Recall: Equation of a circle with centre (0,0) and radius 10 is

$x^2 + y^2 = 100$

Solve for y in terms of x

$y^2 = 100 - x^2$

$y = \pm\sqrt{100 - x^2}$

Note: $+\sqrt{100-x^2}$ is top half

Maximize the area A in $units^2$

$A = lw$

$A = 2x \cdot y$

$A = 2x \cdot \sqrt{100 - x^2}$

$A = 2x \cdot (100 - x^2)^{\frac{1}{2}}$

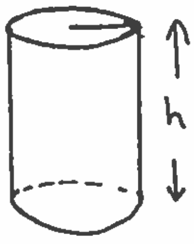
$\frac{dA}{dx} = \ ?$ "use product rule & simplify"

For max. area

$\frac{dA}{dx} = 0$

Note: $0 \le x \le 10$

x	A
0	
?	
10	



Find h in terms of r

$V = 1000 \text{ cm}^3$

$\pi r^2 h = 1000$

$h = \dfrac{1000}{\pi r^2}$

Minimize the surface area in cm^2, S.A

$S.A = 2\pi r^2 + 2\pi r h$

$A_{top\ \&\ bottom}$ $\quad$ A_{sides}

$S.A = 2\pi r^2 + 2\pi r\left(\dfrac{1000}{\pi r^2}\right)$

$S.A = 2\pi r^2 + 2000r^{-1}$

$\dfrac{dS.A}{dr} = ?$

For min S.A

$\dfrac{dS.A}{dr} = 0$

$r^3 = ?$

$r =$

Note: $\dots = \sqrt{\pi}$

If $h = 4$ cm

$\pi r^2(4) = 1000$

$r^2 = \dfrac{250}{\pi}$

$r = \sqrt{\dfrac{250}{\pi}}$

$r \doteq 8.92$

r	S.A
2	
?	
$\sqrt{\frac{250}{\pi}}$	

b) $\dfrac{h}{d}$

$= \dfrac{\frac{1000}{\pi r^2}}{2r}$

$= \dfrac{1000}{\pi r^2} \times \dfrac{1}{2r}$

$= \dfrac{500}{\pi r^3}$

$= \dfrac{500}{\pi(\quad)}$

$= ?$

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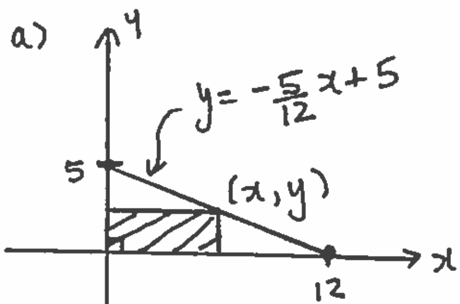
8. a)



Find y in terms of x

$m = \dfrac{-5}{12}$ $\quad b = 5$

$\therefore y = -\dfrac{5}{12}x + 5$

b) see a)

Maximize the area, A in cm^2

$A = lw$

$A = x \cdot y$

$A = x\left(-\dfrac{5}{12}x + 5\right)$

$A = -\dfrac{5}{12}x^2 + 5x$

$\dfrac{dA}{dx} = ?$

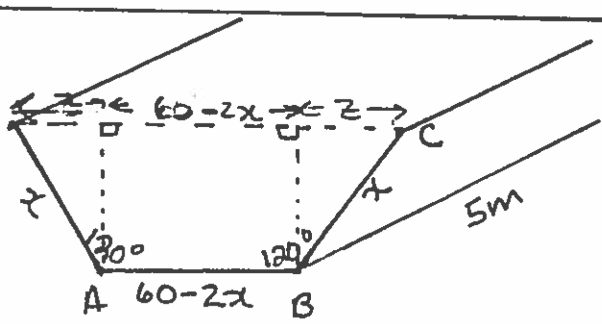
For max. area

$\dfrac{dA}{dx} = 0$

Note: $0 \leq x \leq 12$

x	A
0	
?	
12	

9.



Find h in terms of x

$\dfrac{h}{x} = \cos 30°$

$h = \dfrac{\sqrt{3}}{2}x$

Find z in terms of x

$\dfrac{z}{x} = \sin 30°$

$z = \dfrac{1}{2}x$

a) Maximize area of the trapezoid cross-section ; A in cm^2

$A = \dfrac{(a+b)(h)}{2}$

$A = \dfrac{\left(\left[\frac{1}{2}x + \frac{1}{2}x + 60 - 2x\right] + \left[60 - 2x\right]\right)\left(\frac{\sqrt{3}}{2}x\right)}{2}$

$A = \dfrac{1}{2}(120 - 3x)\left(\dfrac{\sqrt{3}}{2}x\right)$

$A = \dfrac{\sqrt{3}}{4}x(120 - 3x)$

$A = 30\sqrt{3}x - \dfrac{3\sqrt{3}}{4}x^2$

$\dfrac{dA}{dx} = ?$

For max. area

$\dfrac{dA}{dx} = 0$

$x = 20$

Note: $0 \leq x \leq 30$

x	A
0	
?	
30	

b) $V = A_{trapezoid} \times 500 \ (cm^3)$

For max. volume, $x = 20$