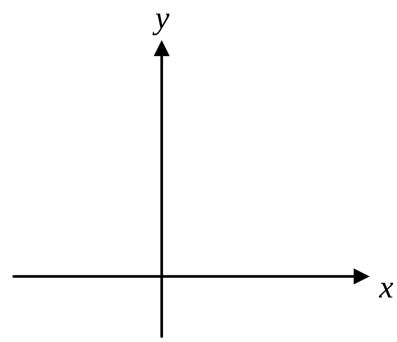
Date:____________ **UNIT 5: DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS**

Section 8.1 – Derivatives of Exponential Functions

Problem: Find the derivative of the exponential function $f(x) = e^x$ from *first principles*.



h	$\frac{e^h - 1}{h}$
0.1	
0.01	
0.001	
0.000 1	

Summary:

If $y = e^x$ then $\frac{dy}{dx} =$

If $y = e^{f(x)}$ then $\frac{dy}{dx} =$

Ex. 1. Find the derivatives of the following.

a) $y = e^{x^2 - x}$

b) $y = \sqrt{e^{\frac{4}{x}}}$

c) $f(x) = \tan 2x \cdot e^{2x}$

d) $s(t) = \frac{e^{3t}}{1 - e^{3t}}$

Ex. 2. Given $f(x) = e^{\sqrt{1-x}}$, determine $f'(-3)$.

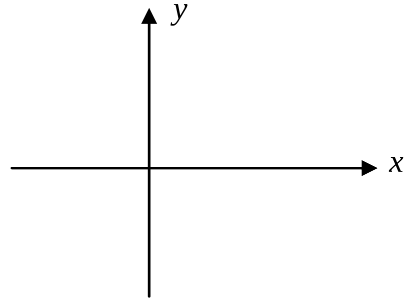
Ex. 3. Find the slope of the normal to the curve $y - e^{xy} = 0$ at the point $(0,1)$.

Ex. 4. Determine the equations of the tangent and normal to $y = \frac{e^x}{x^2}$, $x \neq 0$, at $x = 2$.

HW: pg. 302 # 1 to 5, 6a, 7 to 10, 12ab, 15, 16

Date:______________ **Section 8.2 – The Derivative of the Natural Logarithmic Function**

Find the derivative of $y = \ln x$ **.**



Summary:

If $y = \ln x$ then $\frac{dy}{dx} =$

If $y = \ln u$ then $\frac{dy}{dx} =$

If $y = \ln f(x)$ then $\frac{dy}{dx} =$

Properties of Natural Logarithms:

i) $\ln 1 =$ **ii)** $\ln e =$ **iii)** $\ln e^x =$ **iv)** $e^{\ln x} =$

Natural Logarithm Laws: $x, y > 0$

1. $\ln(x \cdot y) =$ ***multiplication***

2. $\ln\left(\frac{x}{y}\right) =$ ***division***

3. $\ln x^n =$ ***power***

Ex. 1. Differentiate each of the following. (In some cases making use of the logarithm laws and exponent laws will make the process of differentiation easier.)

a) $f(x) = \ln(\cos x)$

b) $y = e^{2\ln x}$

c) $y = \ln x^5$

d) $y = e^{\sqrt{x}} \cdot \ln\sqrt{x}$

e) $h(x)=\dfrac{\ln x}{x^3}$

f) $s(t)=\ln\sqrt{\dfrac{1-t}{1+t}}$

Ex. 2. Find an equation for the normal line to the curve $y\cdot\ln x=e^x-y$ at the point $(1,e)$.

Ex. 3. If $f(x) = x(\ln x)^2$, determine:

a) $f'(x)$

b) the equation of the tangent at $x = \frac{1}{e}$

c) all points at which the graph of $f(x)$ has a horizontal tangent line.

HW: p. 309 # 1, 3, 4, 5ab, 6, 7a, 8, 9a, 10, 14, 15

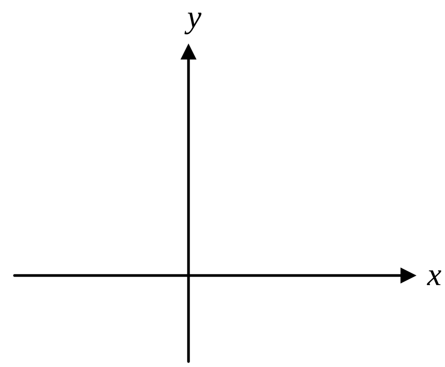
Date:________________

Section 8.3 – Derivatives of the General Exponential and Logarithmic Functions

1. Find the derivative of $y = b^{f(x)}$.

If $y = b^{x}$ then $\frac{dy}{dx} =$

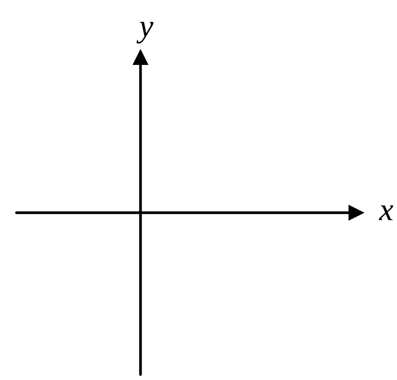
If $y = b^{f(x)}$ then $\frac{dy}{dx} =$



2. Find the derivative of $y = \log_b f(x)$.

If $y = \log_b x$ then $\frac{dy}{dx} =$

If $y = \log_b f(x)$ then $\frac{dy}{dx} =$



Ex. 1. Find the derivative of each of the following.

a) $y = e^x + x^e - \ln x$

b) $y = 3^x + x^3 - \log_3 x$

c) $s = 10^{\sqrt{t^2-1}}$

d) $f(x) = \log_4 \sqrt{3x^2 - 5}$

e) $y = \log\left(\frac{1-x}{1+x}\right)$

Ex. 2. Find the equation of the tangent to $g(x) = x^3 \cdot \log_2 x^3$ at $x = 1$.

HW: p. 316 # 1 to 4, 5a, 6, 7ab

Date:______________ **Section 8.5 – Logarithmic Differentiation**

Warm-up: Find the slope of the tangent to $y=\dfrac{(x^4+1)\sqrt{x+2}}{\sqrt[3]{2x^2+2x+1}}$ at $x=-1$.

Recall: If $y=x^n$ then $\dfrac{dy}{dx}=$

If $y=e^x$ then $\dfrac{dy}{dx}=$

If $y=b^x$ then $\dfrac{dy}{dx}=$

If $y=\ln x$ then $\dfrac{dy}{dx}=$

If $y=\log_b x$ then $\dfrac{dy}{dx}=$

If $y=[f(x)]^n$ then $\dfrac{dy}{dx}=$

If $y=e^{f(x)}$ then $\dfrac{dy}{dx}=$

If $y=b^{f(x)}$ then $\dfrac{dy}{dx}=$

If $y=\ln f(x)$ then $\dfrac{dy}{dx}=$

If $y=\log_b f(x)$ then $\dfrac{dy}{dx}=$

Ex. 1. Differentiate each of the following.

a) $y=\pi x+x^{\pi}-\pi^{x}$

b) $f(x)=2x^{2\sqrt{3}}-\log_2 x$

Note: If $y=x^{f(x)}$ or if $y=[f(x)]^{g(x)}$, the derivative is found by first taking the natural logarithm of both sides and then differentiating with respect to *x*.
This is the technique of ***logarithmic differentiation***.

Ex. 2. Use the technique of ***logarithmic differentiation*** to find the derivative for each of the following:

a) $f(x)=x^x$

b) $y=(\cos x)^{\sin x}$

Ex. 3. If $y = (x^2 - 1)^{\sqrt{x}}$ find $\frac{dy}{dx}$.

Ex. 4. Find the equation of the normal to $y = \frac{(x^4 + 1)\sqrt{x+2}}{\sqrt[3]{2x^2 + 2x + 1}}$ at $x = -1$,

using ***logarithmic differentiation***.

HW: p. 326 # 1, 3b, 2, 3ac, 4, 6

Date:_______________

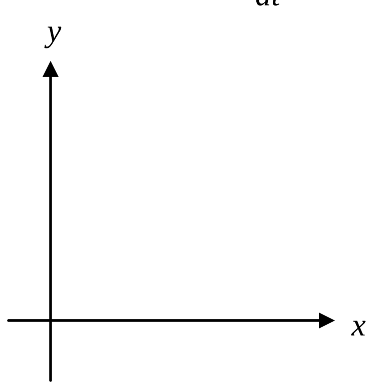
Applications of Exponential and Logarithmic Functions

Ex. 1. The position of a particle that moves on a straight line is given by $s(t) = t^{\frac{1}{t}}$ for $t > 0$.

a) Find the velocity.

b) At what time, *t*, is the velocity zero?

Ex. 2. A playground slide has the shape of the curve $y = 4e^{-\frac{x}{2}}$. If the horizontal component of the velocity for a person on the slide is $\frac{dx}{dt} = 2\ m/s$ when $x = 1$, find the vertical component of the velocity $\frac{dy}{dt}$ at that instant.



Ex. 3. Find the maximum and minimum values of $f(x) = \ln(e^x + 4e^{-x})$ for $-\ln 2 \le x \le \ln 4$.

x	$f(x)$

Ex. 4. The effectiveness of studying for a test depends on how many hours a student studies. Some experiments showed that if the effectiveness, *E*, is put on a scale of 0 to 10, then $E(t) = 0.5\left[10 + te^{-\frac{t}{20}}\right]$, where *t* is the number of hours spent studying for an examination.

If a student has up to 30 h that he can spend studying, how many hours should he study for maximum effectiveness?

t	$E(t)$

HW: p. 303 #13, 14; p. 310 #11 to 13; p. 316 #8; p. 320 #2a., 4, 7
REVIEW for TEST: p. 330 #1 to 13