Date: May 5/14

UNIT 7 - VECTORS

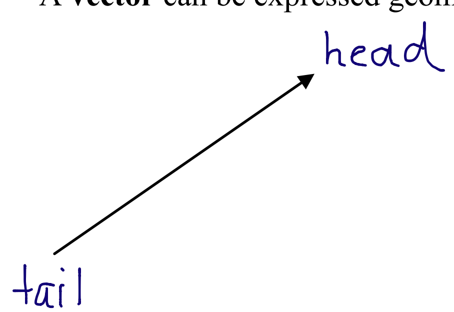
Section 4.1 - VECTOR CONCEPTS

Definition: A **scalar** is a quantity that has magnitude only.

A **vector** is a quantity that has magnitude and direction.

Scalar Quantity	Vector Quantity
1. distance Maya lives 100 km from Kitchener.	**1. displacement** Maya lives 100 km northeast of Kitchener.
2. speed The jet is travelling at 900 km/h.	**2. velocity** The jet is travelling at 900 km/h west.
3. mass Joe has a mass of 100 kg.	**3. weight** Joe has a weight of 980 N (downwards).

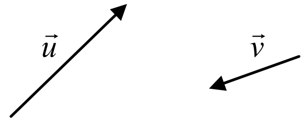
A **vector** can be expressed geometrically by a ***directed line segment***.



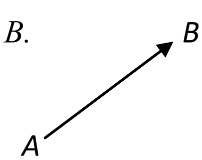
- ***"directed "*** means that one end has been designated the ***tail*** and the other end the ***head***.
- the direction is from its tail to its head. and is designated by an **arrow**.

The ***notation*** used to describe vector quantities is as follows:

i) $\vec{u}$ and $\vec{v}$ are vectors.



iii) $\overrightarrow{AB}$ is the vector that starts at point A and ends at point B.

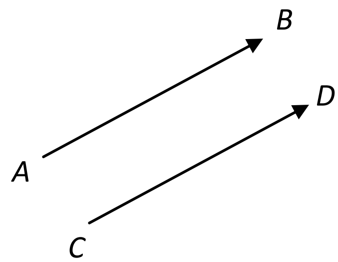


ii) $|\vec{u}|$ and $|\vec{v}|$ are the magnitudes of the vectors $\vec{u}$ and $\vec{v}$.

iv) $|\overrightarrow{AB}|$ is the magnitude of vector $\overrightarrow{AB}$.

Definitions:

Equal Vectors: Two ***vectors*** are ***equal*** if and only if their ***magnitudes*** and their ***directions*** are the ***same***.

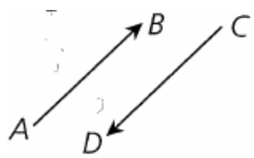


If $\overrightarrow{AB} = \overrightarrow{CD}$
then
i) $|\overrightarrow{AB}| = |\overrightarrow{CD}|$
ii) same directions

If i) $|\overrightarrow{AB}| = |\overrightarrow{CD}|$
& ii) same directions
then
$\overrightarrow{AB} = \overrightarrow{CD}$

Opposite Vectors: Two ***vectors*** are ***opposite*** if they have the ***same magnitude*** but ***opposite directions***.

When two ***vectors*** are ***opposite***, such as $\overrightarrow{AB}$ and $\overrightarrow{CD}$,
one is the ***negative*** of the other: $\overrightarrow{AB} = -\overrightarrow{CD}$ or $\overrightarrow{CD} =$ $-\overrightarrow{AB}$



Note: $-\overrightarrow{CD} = \overrightarrow{DC}$ and $-\overrightarrow{AB} =$ $\overrightarrow{BA}$

Parallel Vectors: Two ***vectors*** are ***parallel*** if their ***directions*** are the ***same*** or ***opposite***.

Zero Vector: The ***zero vector***, $\vec{0}$, has a ***magnitude*** of 0 and a ***direction*** that is ***undefined***.

$\vec{0}$ •

Unit Vector: A ***unit vector***, $\hat{v}$, has a ***magnitude*** of 1 unit, ie. $|\hat{v}| = 1$.

1. A unit vector in the direction of any vector $\vec{v}$ can be found by dividing $\vec{v}$ by its magnitude $|\vec{v}|$.

$$\hat{v} = \boxed{\frac{1}{|\vec{v}|}}\vec{v}$$

Scalar

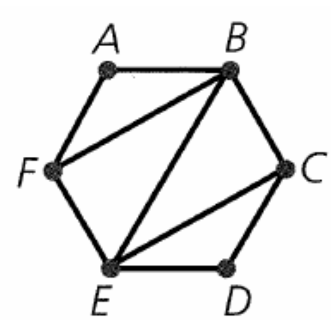
2. Any vector $\vec{v}$ can be expressed as the product of its magnitude $|\vec{v}|$ and a unit vector $\hat{v}$ in the direction of $\vec{v}$.

$$\vec{v} = \boxed{|\vec{v}|}\,\hat{v}$$

Scalar

Ex. 1. *ABCDEF* is a regular hexagon. Find two vectors which are:

a) equal
b) parallel but having different magnitudes
c) equal in magnitude but opposite in direction



a) $\overrightarrow{AB} = \overrightarrow{ED}$
$\overrightarrow{FE} = \overrightarrow{BC}$

b) $\overrightarrow{AF}$ & $\overrightarrow{BE}$
$\overrightarrow{DC}$ & $\overrightarrow{EB}$

c) $\overrightarrow{FB}$ & $\overrightarrow{CE}$
$\overrightarrow{FA}$ & $\overrightarrow{CD}$

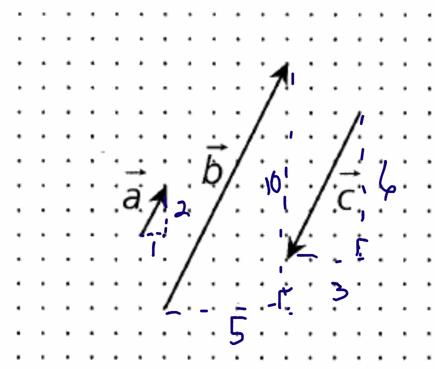
Ex. 2. a) Express $\vec{b}$ and $\vec{c}$ each as a scalar multiple of $\vec{a}$.

$|\vec{a}|^2 = 1^2 + 2^2$ $\quad |\vec{b}|^2 = 5^2 + 10^2$ $\quad |\vec{c}|^2 = 3^2 + 6^2$

$|\vec{a}| = \sqrt{5}$ $\quad |\vec{b}| = \sqrt{125}$ $\quad |\vec{c}| = \sqrt{45}$

$|\vec{b}| = 5\sqrt{5}$ $\quad |\vec{c}| = 3\sqrt{5}$

$\therefore \vec{b} = 5\vec{a}$ & $\vec{c} = -3\vec{a}$

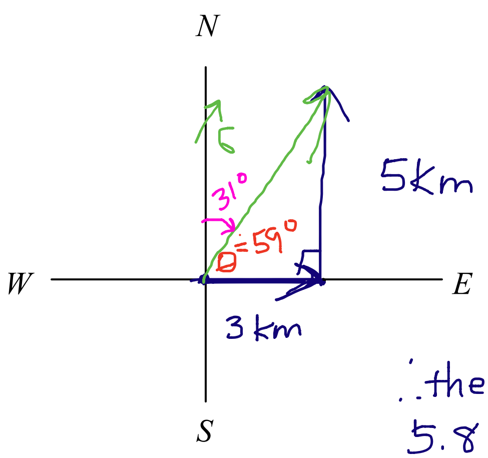


b) Express $\vec{a}$, $\vec{b}$ and $\vec{c}$ each in terms of the unit vector $\hat{a}$.

$\hat{a} = \frac{1}{|\vec{a}|}\vec{a}$ $\therefore \vec{a} = \sqrt{5}\,\hat{a}$, $\vec{b} = 5\sqrt{5}\,\hat{a}$ & $\vec{c} = -3\sqrt{5}\,\hat{a}$

$= \underbrace{\frac{1}{\sqrt{5}}\vec{a}}$

Ex. 3. A driver travels 3 km due east and then 5 km due north. Find the ***resultant vector***, $\vec{r}$.
scale: *1 cm = 2 km*



Find $|\vec{r}|$

$|\vec{r}| = \sqrt{3^2 + 5^2}$

$|\vec{r}| = \sqrt{34}$

$|\vec{r}| \doteq 5.8$

Find θ

$\tan\theta = \frac{5}{3}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right)$

$\theta \doteq 59°$

$\therefore$ the resultant vector is about 5.8 km [N 31° E] or [E 59° N]

Recall:

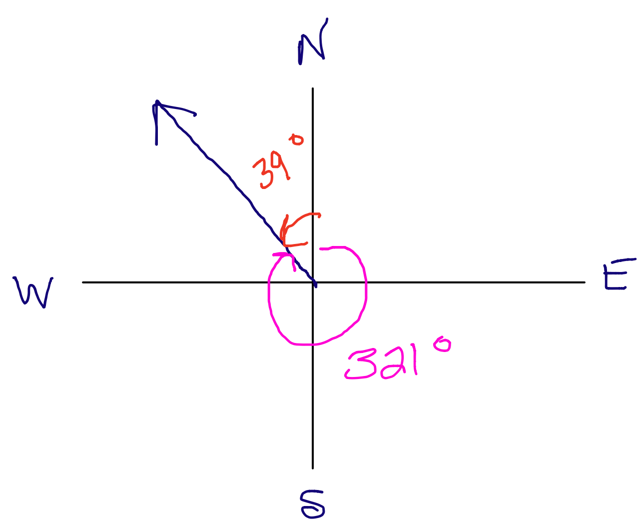
"Bearings" are used to describe direction as:

i) an angle measured by compass bearings.

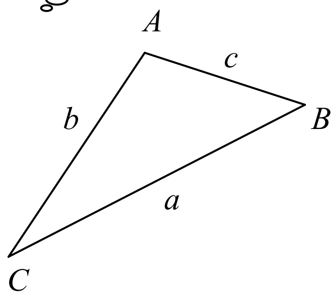
ii) an angle from the north and measured in a clockwise direction.

Ex. 4. Express $N39°W$ as a bearing.

$\therefore$ N39°W is a bearing of 321°.



Recall:



THE COSINE LAW

For SAS, find S

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

For SSS, find A

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

THE SINE LAW

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

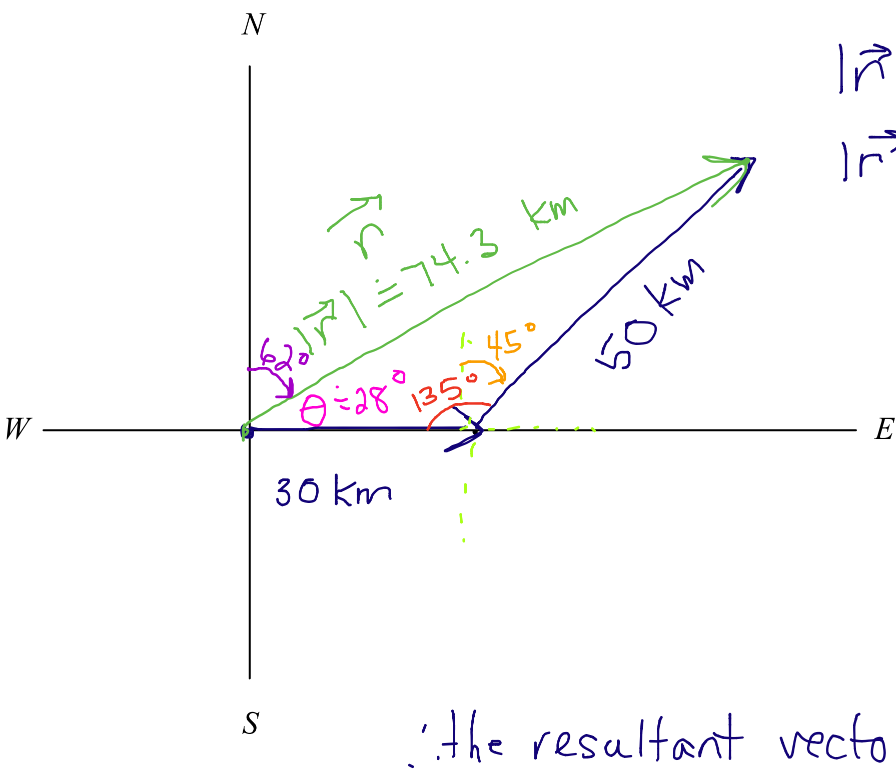
or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

NE

Ex. 5. Mary drives 30 km due east and then 50 km $N45^\circ E$.
Find the ***resultant vector***, $\vec{r}$.

scale: *1 cm = 10 km*



Find $|\vec{r}|$

$|\vec{r}|^2 = 30^2 + 50^2 - 2(30)(50)\cos 135^\circ$

$|\vec{r}| \doteq 74.3$

Find θ

$\dfrac{\sin\theta}{50} = \dfrac{\sin 135^\circ}{74.3}$

$\sin\theta \doteq 0.4758$

$\theta = \sin^{-1}(0.4758)$

$\theta \doteq 28^\circ$

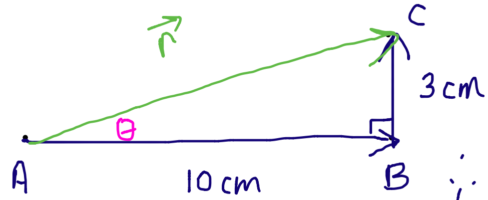
$\therefore$ the resultant vector is about 74.3 km [N62° E]

HW: pg. 127 #1, 2, 4 to 13

Date: May 6/14

Section 4.2 – Vector Laws – Geometrically

Ex. 1. Consider a particle at a point *A* and suppose that the particle is displaced 10 cm to the right and then 3 cm upwards. Determine the resultant vector.



Find $|\vec{r}|$

$|\vec{r}| = \sqrt{10^2+3^2}$

$|\vec{r}| = \sqrt{109}$

$|\vec{r}| \doteq 10.4$

Find θ

$\tan\theta = \frac{3}{10}$

$\theta = \tan^{-1}\left(\frac{3}{10}\right)$

$\theta \doteq 17°$

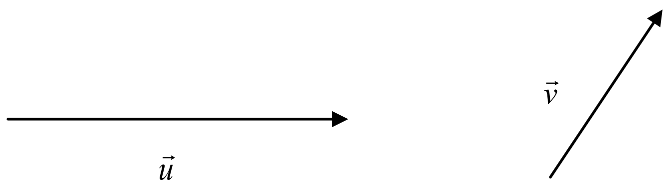
$\therefore$ the resultant vector is about 10.4 cm at approximately 17° to the horizontal.

Note: In **Ex. 1.** the two displacements may be represented by vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

So, $\overrightarrow{AB} + \overrightarrow{BC}$ gives the resultant $\overrightarrow{AC}$.

VECTOR ADDITION:

Suppose you are given two vectors $\vec{u}$ and $\vec{v}$, below.

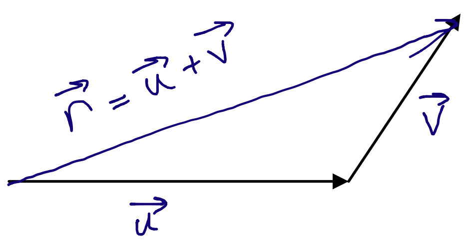


There are two ways of adding vectors geometrically.

I <u>Triangle Law of Addition</u>

Find the sum $\vec{u}+\vec{v}$.

- translate the vectors so the "tail" of $\vec{v}$ is at the "head" of $\vec{u}$.
  ie. "head-to-tail"

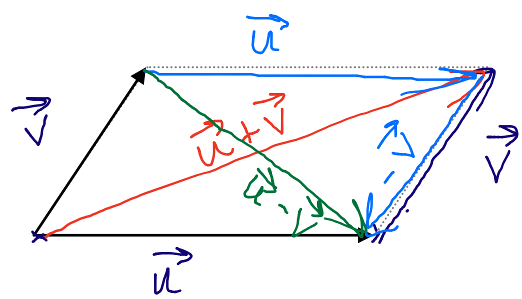


- the resultant vector is the vector from the "tail" of $\vec{u}$ to the "head" of $\vec{v}$.

II <u>Parallelogram Law of Addition</u>

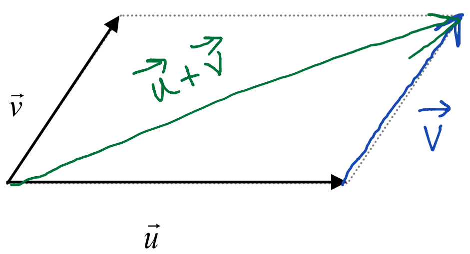
Find the sum $\vec{u}+\vec{v}$ and difference $\vec{u}-\vec{v}$.

- translate $\vec{u}$ and $\vec{v}$ so that they are "tail-to-tail".



- complete the parallelogram
- the sum $\vec{u}+\vec{v}$ is the diagonal of the parallelogram that originates at the "tails".
- the difference $\vec{u}-\vec{v}$ is the vector created by the second diagonal.

How does the $|\vec{u}+\vec{v}|$ compare to $|\vec{u}|$ and $|\vec{v}|$?



Note:

$|\vec{u}+\vec{v}| < |\vec{u}| + |\vec{v}|$

&

$|\vec{u}+\vec{v}| = |\vec{u}| + |\vec{v}|$ if $\vec{u}$ and $\vec{v}$ are colinear.

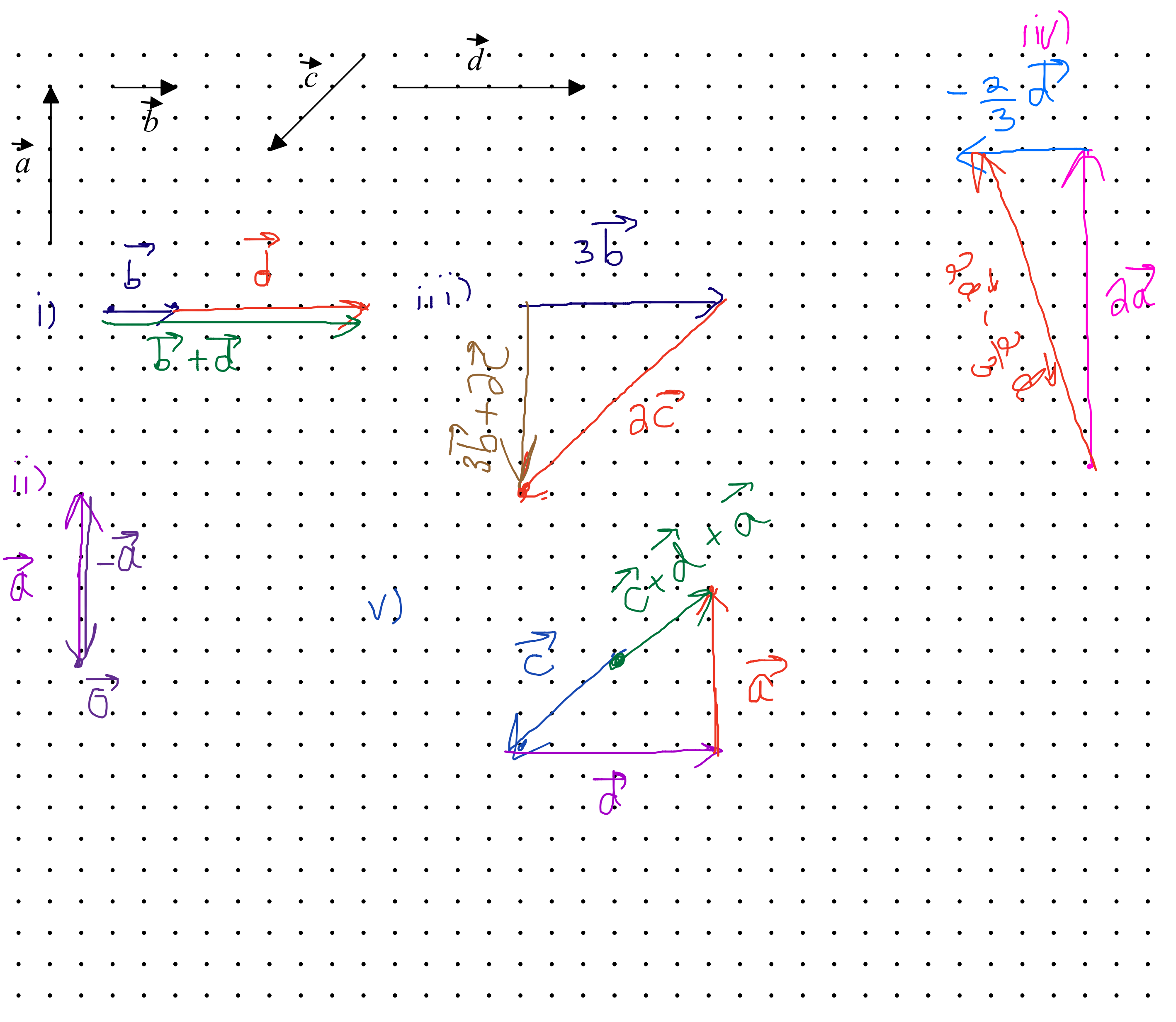
generally, $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

Triangle Inequality

For vectors $\vec{u}$ and $\vec{v}$, $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

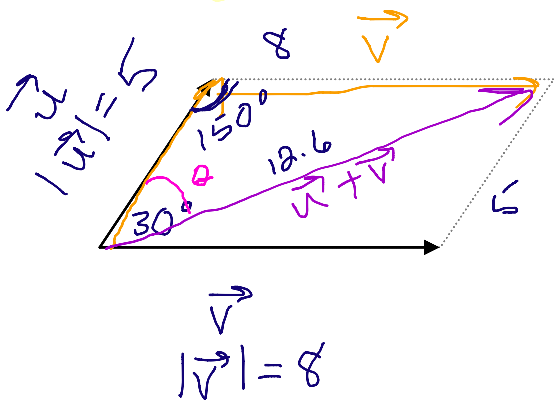
Ex. 2. Given $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ below, construct the following:

i) $\vec{b}+\vec{d}$ **ii)** $\vec{a}+(-\vec{a})$ **iii)** $3\vec{b}+2\vec{c}$ **iv)** $2\vec{a}-\frac{2}{3}\vec{d}$ **v)** $\vec{c}+\vec{d}+\vec{a}$



Ex. 3. Find the magnitude and direction of the sum of two vectors $\vec{u}$ and $\vec{v}$, if their magnitudes are 5 and 8 respectively, and the angle between them is 30°. Find $\underbrace{\vec{u}+\vec{v}}$.
Note: The angle between their tails is 30°.

magnitude & direction



Find $|\vec{u}+\vec{v}|$

$|\vec{u}+\vec{v}|^2 = 5^2 + 8^2 - 2(5)(8)\cdot\cos 150^\circ$

$|\vec{u}+\vec{v}| \doteq 12.6$

Find θ

$\cos\theta = \dfrac{5^2 + 12.6^2 - 8^2}{2(5)(12.6)}$

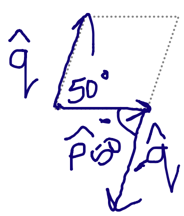
$\cos\theta \doteq 0.9505$

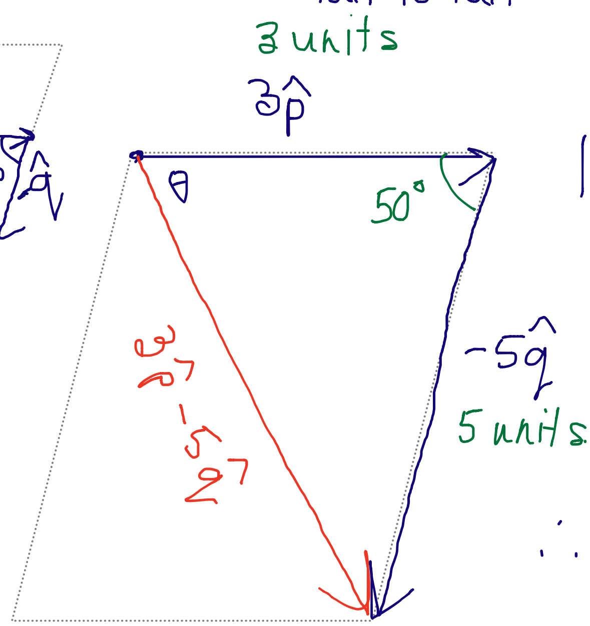
$\theta \doteq 18^\circ$

$\therefore$ $\vec{u}+\vec{v}$ is about 12.6 units, at an angle of 18° to $\vec{u}$ or 12° to $\vec{v}$.

Ex. 4. $\hat{p}$ and $\hat{q}$ are unit vectors that make an angle of 50° with each other. Find $|3\hat{p}-5\hat{q}|$.

"tail to tail"





Find $|3\hat{p}-5\hat{q}|$

$|3\hat{p}-5\hat{q}|^2 = 3^2 + 5^2 - 2(3)(5)\cdot\cos 50^\circ$

$|3\hat{p}-5\hat{q}| \doteq 3.8$

$\therefore |3\hat{p}-5\hat{q}| \doteq 3.8$ units.

HW: pg. 133 #1, 2, 4 to 9, 14, 15, 18 to 21 **Note:** Answer for #**5** should be 37.7 km E15°S