Date:_____________

UNIT 7 – AN INTRODUCTION TO VECTORS

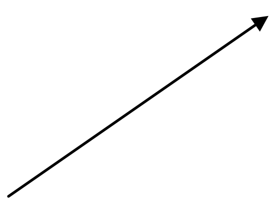
Section 4.1 - VECTOR CONCEPTS

Definition: A **scalar** is a quantity that has ______________________ only.

A **vector** is a quantity that has _______________________ and _______________________.

Scalar Quantity	Vector Quantity
1. distance Maya lives 100 km from Kitchener.	**1. displacement** Maya lives 100 km northeast of Kitchener.
2. speed The jet is travelling at 900 km/h.	**2. velocity** The jet is travelling at 900 km/h west.
3. mass Joe has a mass of 100 kg.	**3. weight** Joe has a weight of 980 N (downwards).

A **vector** can be expressed geometrically by a ***directed line segment***.

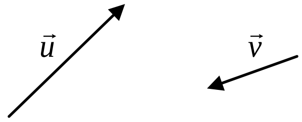


- "***directed*** " means that one end has been designated the ***tail*** and the other end the ***head***.

- the direction is from its ____________ to its ___________ and is designated by an **arrow**.

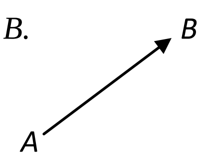
The ***notation*** used to describe vector quantities is as follows:

i) $\vec{u}$ and $\vec{v}$ are vectors.



ii) $|\vec{u}|$ and $|\vec{v}|$ are the magnitudes of the vectors $\vec{u}$ and $\vec{v}$.

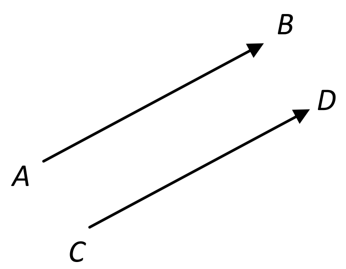
iii) $\overrightarrow{AB}$ is the vector that starts at point *A* and ends at point *B*.



iv) $|\overrightarrow{AB}|$ is the magnitude of vector $\overrightarrow{AB}$.

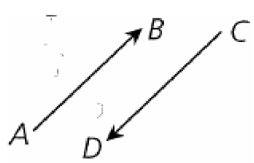
Definitions:

Equal Vectors: Two ***vectors*** are ***equal*** if and only if their ***magnitudes*** and their ***directions*** are the ***same***.



Opposite Vectors: Two ***vectors*** are ***opposite*** if they have the ***same magnitude*** but ***opposite directions***.

When two ***vectors*** are ***opposite***, such as $\overrightarrow{AB}$ and $\overrightarrow{CD}$, one is the ***negative*** of the other: $\overrightarrow{AB} = -\overrightarrow{CD}$ or $\overrightarrow{CD} =$ ________



Note: $-\overrightarrow{CD} = \overrightarrow{DC}$ and $-\overrightarrow{AB} =$ ________

Parallel Vectors: Two ***vectors*** are ***parallel*** if their ***directions*** are the ***same*** or ***opposite***.

Zero Vector: The ***zero vector***, $\vec{0}$, has a ***magnitude*** of 0 and a ***direction*** that is ***undefined***.

Unit Vector: A ***unit vector***, $\hat{v}$, has a ***magnitude*** of 1 unit, ie. $|\hat{v}| = 1$.

1. A unit vector in the direction of any vector $\vec{v}$ can be found by dividing $\vec{v}$ by its magnitude $|\vec{v}|$.

$$\hat{v} = \frac{1}{|\vec{v}|}\vec{v}$$

2. Any vector $\vec{v}$ can be expressed as the product of its magnitude $|\vec{v}|$ and a unit vector $\hat{v}$ in the direction of $\vec{v}$.

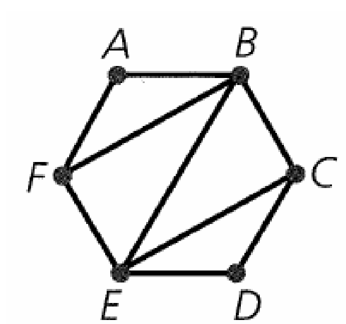
$$\vec{v} = |\vec{v}|\,\hat{v}$$

Ex. 1. *ABCDEF* is a regular hexagon. Find two vectors which are:

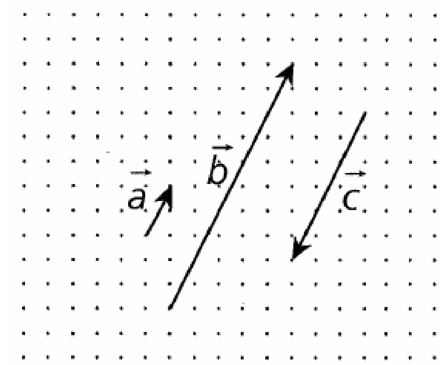
a) equal

b) parallel but having different magnitudes

c) equal in magnitude but opposite in direction

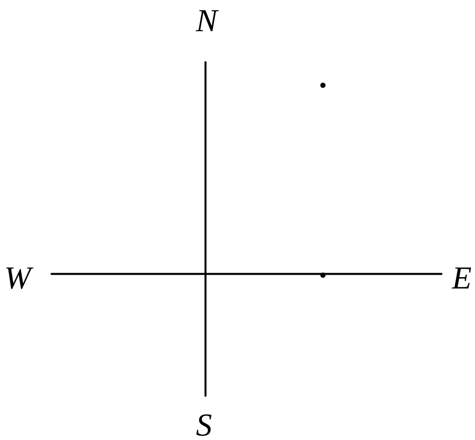


Ex. 2. a) Express $\vec{b}$ and $\vec{c}$ each as a scalar multiple of $\vec{a}$.



b) Express $\vec{a}$, $\vec{b}$ and $\vec{c}$ each in terms of the unit vector $\vec{a}$.

Ex. 3. A driver travels 3 km due east and then 5 km due north. Find the ***resultant vector***, $\vec{r}$.
scale: *1 cm = 2 km*

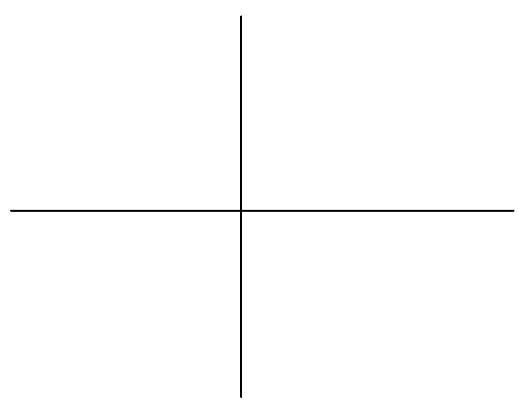


*"**Bearings**"* are used to describe direction as:

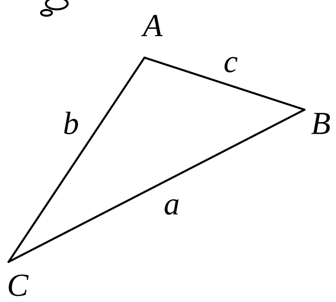
i) an angle measured by compass bearings.

ii) an angle from the north and measured in a clockwise direction.

Ex. 4. Express $N39^\circ W$ as a bearing.



***Recall*:**



THE COSINE LAW

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

THE SINE LAW

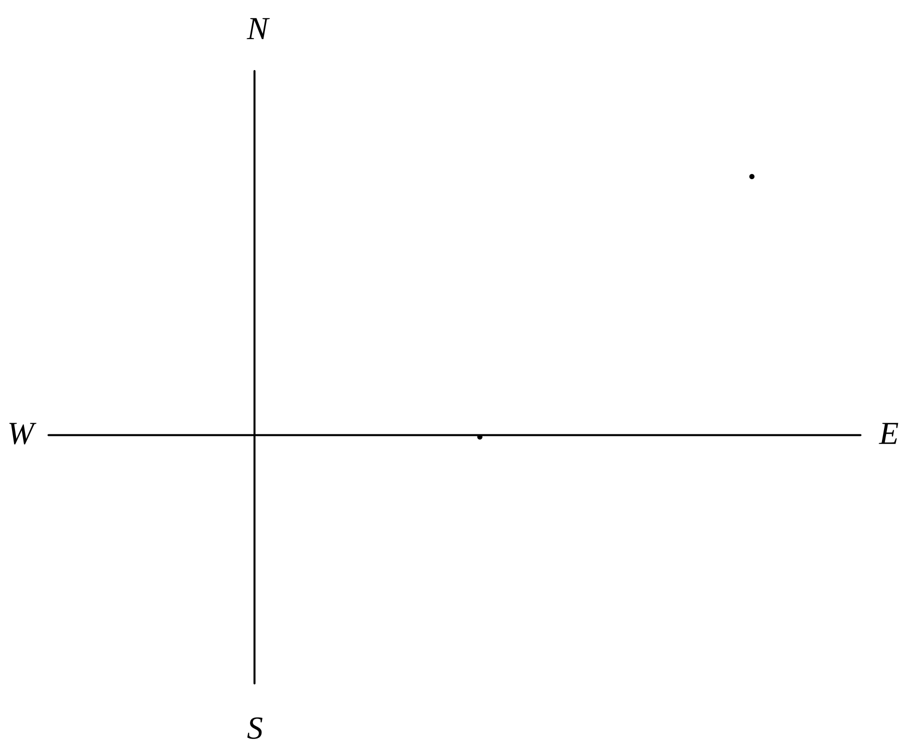
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex. 5. Mary drives 30 km due east and then 50 km $N45^\circ E$.
Find the ***resultant vector***, $\vec{r}$.

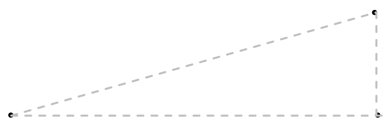
scale: *1 cm = 10 km*



HW: p. 127 #1, 2, 4 to 13

Date:______________ **Section 4.2 – Vector Laws – Geometrically**

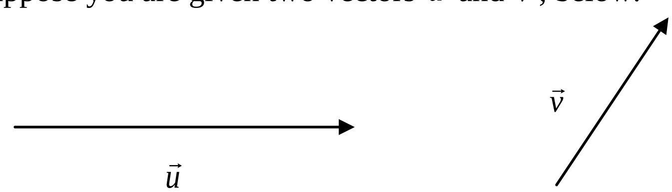
Ex. 1. Consider a particle at a point *A* and suppose that the particle is displaced 10 cm to the right and then 3 cm upwards. Determine the resultant vector.



Note: In **Ex. 1.** the two displacements may be represented by vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
So, $\overrightarrow{AB} + \overrightarrow{BC}$ gives the resultant ________ .

VECTOR ADDITION:

Suppose you are given two vectors $\vec{u}$ and $\vec{v}$, below.



There are two ways of adding vectors geometrically.

I **<u>Triangle Law of Addition</u>**

Find the sum $\vec{u}+\vec{v}$.

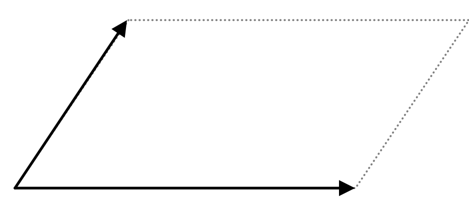
- translate the vectors so the "tail" of $\vec{v}$ is at the "head" of $\vec{u}$.
ie. "head-to-tail"

- the resultant vector is the vector from the "tail" of $\vec{u}$ to the "head" of $\vec{v}$.

II **<u>Parallelogram Law of Addition</u>**

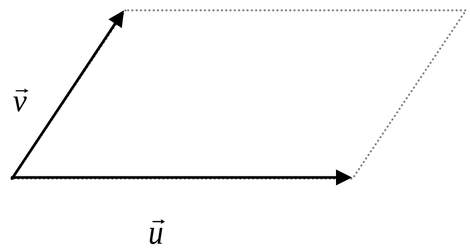
Find the sum $\vec{u}+\vec{v}$ and difference $\vec{u}-\vec{v}$.

- translate $\vec{u}$ and $\vec{v}$ so that they are "tail-to-tail".



- complete the parallelogram
- the sum $\vec{u}+\vec{v}$ is the diagonal of the parallelogram that originates at the "tails".
- the difference $\vec{u}-\vec{v}$ is the vector created by the second diagonal.

How does the $|\vec{u}+\vec{v}|$ compare to $|\vec{u}|$ and $|\vec{v}|$?



Note:

$|\vec{u}+\vec{v}| < |\vec{u}| + |\vec{v}|$

&

$|\vec{u}+\vec{v}| = |\vec{u}| + |\vec{v}|$ if $\vec{u}$ and $\vec{v}$ are ______________________.

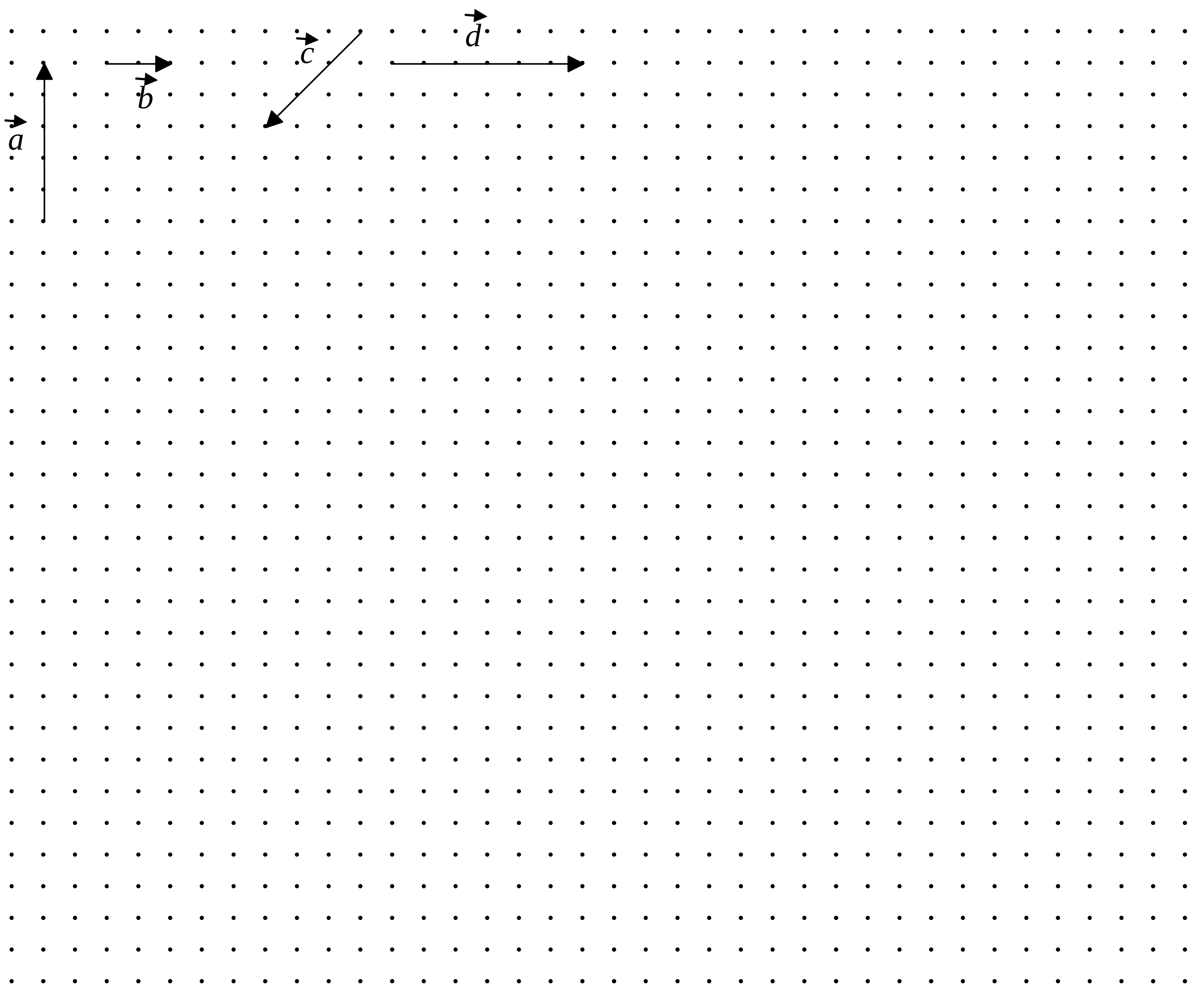
generally, $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

Triangle Inequality

For vectors $\vec{u}$ and $\vec{v}$, $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

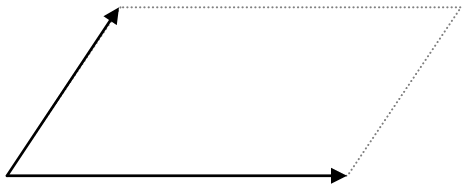
Ex. 2. Given $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ below, construct the following:

i) $\vec{b}+\vec{d}$ **ii)** $\vec{a}+(-\vec{a})$ **iii)** $3\vec{b}+2\vec{c}$ **iv)** $2\vec{a}-\frac{2}{3}\vec{d}$ **v)** $\vec{c}+\vec{d}+\vec{a}$

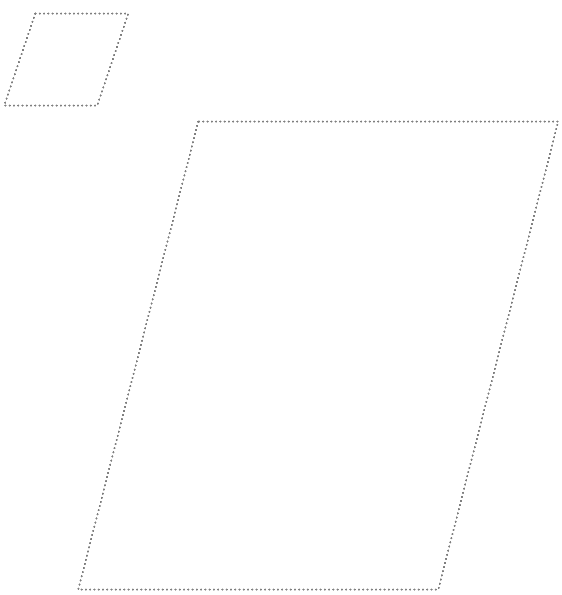


Ex. 3. Find the magnitude and direction of the sum of two vectors $\vec{u}$ and $\vec{v}$, if their magnitudes are 5 and 8 respectively, and the angle between them is 30°. Find $\vec{u}+\vec{v}$.

Note: The angle between their tails is 30°.



Ex. 4. $\hat{p}$ and $\hat{q}$ are unit vectors that make an angle of 50° with each other. Find $|3\hat{p}-5\hat{q}|$.



HW: p. 133 #1, 2, 4 to 9, 14, 15, 18 to 21 **Note:** Answer for #**5** should be 37.7 km E15° S

Date:_____________ **Section 4.2 – Vector Laws – Algebraically**

Recall: In **Ex. 1.** from the previous day's note we learned **geometrically** that

$\overrightarrow{AB} + \overrightarrow{BC}$

=

Today we will learn how to add and subtract vectors **algebraically**.

Ex. 1. Add and or subtract the following vectors algebraically.

a) $\overrightarrow{PQ} + \overrightarrow{QT}$

b) $\overrightarrow{ST} - \overrightarrow{UT}$

c) $\overrightarrow{BA} + \overrightarrow{TB} + \overrightarrow{QT} + \overrightarrow{PQ}$

d) $\overrightarrow{AD} - \overrightarrow{BD} - \overrightarrow{AC} - \overrightarrow{CB}$

Properties of Vector Algebra

Given $\vec{a}$, $\vec{b}$ and $\vec{c}$ are distinct non-zero vectors and *k* and *m* are real numbers, then

A. Properties of Vector Addition

***1.** $\vec{a} + \vec{b} =$
Commutative Law

***2.** $(\vec{a} + \vec{b}) + \vec{c} =$
Associative Law

3. $\vec{a} + \vec{0} =$

4. $\vec{a} + (-\vec{a}) =$

B. Properties of Scalar Multiplication

1. $(km)\vec{a} =$
Associative Law

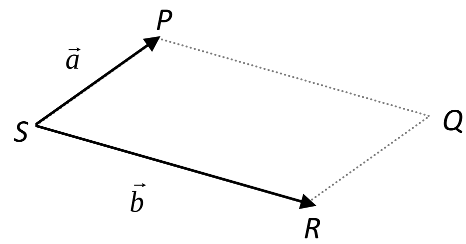
***2.** $k(\vec{a} + \vec{b}) =$
Distributive Law

3. $(k + m)\vec{a} =$
Distributive Law

***** We will prove these vector properties.

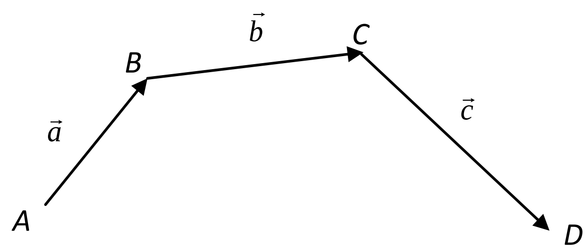
A. 1. Prove: $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ **(Commutative Law for Vector Addition)**

Proof: *Let* $\vec{a}$ *and* $\vec{b}$ *be distinct non-zero vectors.*



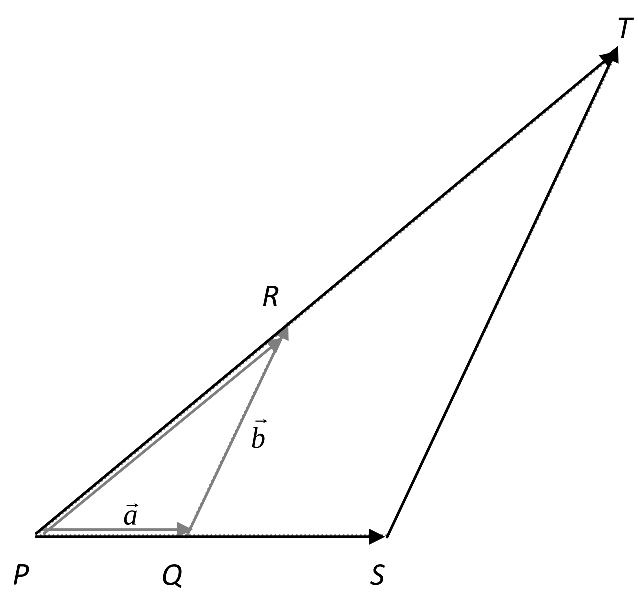
A. 2. Prove: $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ **(Associative Law for Vector Addition)**

Proof: *Let* $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ *be distinct non-zero vectors.*



B. 2. Prove: $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$ **(Distributive Law for Scalar Multiplication)**

Proof: *Let* $\vec{a}$ *and* $\vec{b}$ *be distinct non-zero vectors.*



Ex. 2. Simplify.

a) $4\vec{v}+3\vec{v}-2\vec{v}$

b) $3\vec{u}+4(\vec{v}-\vec{u})$

c) $4(\vec{a}+\vec{b})+3(\vec{a}-\vec{b})$

d) $2(\vec{b}-\vec{a})-5(\vec{b}+\vec{a})$

Ex. 3. If $\vec{u}=2\vec{a}-3\vec{b}$ and $\vec{v}=4\vec{a}+5\vec{b}$ then express $\vec{a}$ and $\vec{b}$ in terms of $\vec{u}$ and $\vec{v}$.

HW: p. 133 #3, 10 to 12, 13 algebraically only

Date:______________ **Section 4.3 – Force as a Vector – Part I**

<u>Force</u> - that which pulls, pushes, compresses, distorts in any way
- that which changes the state of rest or state of motion in a body
- **note:** the magnitude of a force is measured in **newtons**, **N**, where a 1 kg object weighs approximately 9.8 N

<u>Vector Force</u> – a vector equal in magnitude to the magnitude of a given force and having its direction parallel to the line of action of the force.

<u>Equilibrant</u> – the force that is equal in magnitude but opposite in direction to the resultant

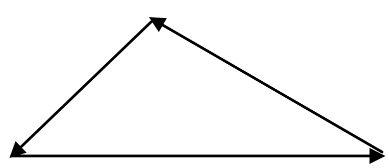
*An object will be in a state of equilibrium when the **resultant** of all the forces acting on it is **zero**. This means that the three given force vectors must form a <u>triangle</u> or be <u>collinear</u> According to the **triangle inequality theorem**, a triangle can only be formed if the sum of the shorter two sides is always greater than the longest side.*

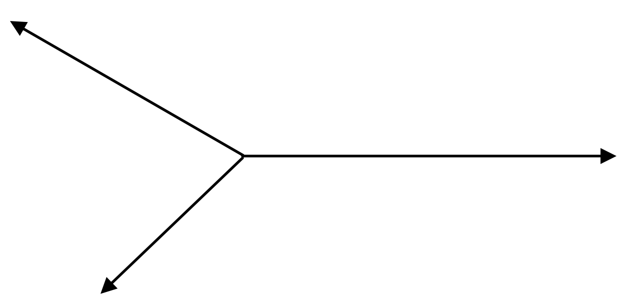
Note: Overall the sum of the smaller two magnitudes must be greater than or equal to the larger magnitude of the forces for equilibrium to be achieved.

Ex. 1. Which of the following sets of forces could keep an object at rest (in a state of equilibrium)?

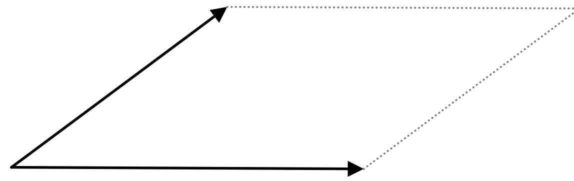
a) 8N, 4N and 12N **b)** 3N, 4N and 12N **c)** 5N, 7N and 10N

Ex. 2. The three forces 5N, 7N and 10N are applied to an object. If the object is in a state of equilibrium, show how the forces must be arranged and calculate the angles between the lines of action of the three forces.





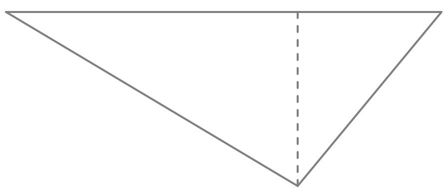
Ex. 3. Find the ***resultant*** and ***equilibrant*** when forces of 48N and 35N act at an angle of 35° to each other.



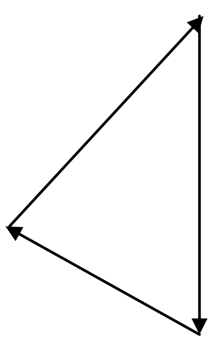
Ex. 4. A 15 kg mass is suspended from the ceiling by two cords that make angles of 30° and 50° with the ceiling. Find the tensions in these cords. **Note:** A 1 kg mass exerts a force of 9.8 N.

Position Diagram

A 15 kg mass exerts a force of



Vector Force Diagram



HW: p. 141 #3, 4, 6, 7, 9 to 13

Date:_______________

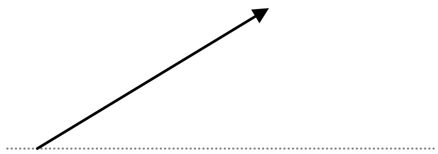
Section 4.3 – Force as a Vector – Part II

Vector Components: A vector can be broken down into ***horizontal*** *and* ***vertical components.***

Ex. 1. A sled is being pulled by a force of 50N. If the rope makes an angle of 31° with the ground, find

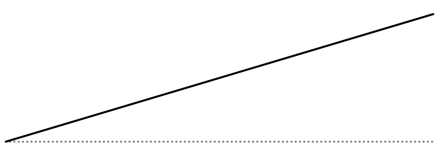
a) the force that is pulling the sled forward (*horizontal component*)

b) the force that is pulling the sled upward (*vertical component*)

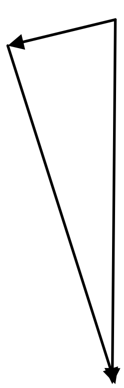


Ex. 2. A 20 kg trunk is resting on a ramp inclined at an angle of 15°. Calculate the components of the force of gravity on the trunk that are parallel and perpendicular to the ramp.

Position Diagram



Vector Force Diagram



HW: p. 141 #2, 15 to 22

Date:_____________

Section 4.4 – Velocity as a Vector

Ex. 1. A boat with a water speed of 5 km/h sets out to cross a 1 km wide river which flows at 2 km/h. A pub lies directly across the river on the opposite bank.

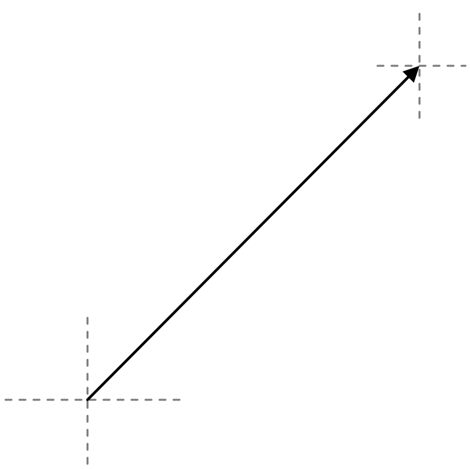
a) If the boat attempts to head straight across, how long will it take to reach the other side and how far downstream will the boat land on the opposite bank?

b) If the boat crosses the river and arrives directly at the pub on the opposite bank, in what direction must the boat steer and how long will it take to cross?

a)

b)

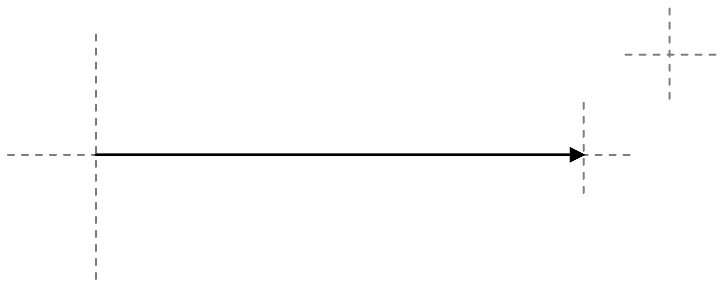
Ex. 2. A pilot is flying her 747 jet at 525 km/h in the direction $N\,45^\circ E$. She encounters a wind from $N\,60^\circ W$ at 98 km/h. Find the resultant velocity.



Ex. 3. The pilot of an airplane that flies at 800 km/h wishes to travel to a city 800 km due east. There is an 80 km/h wind from the northeast.

a) What should the plane's heading be?

b) How long will the trip take?



HW: p. 149 #2a, 4 to 10;

REVIEW for TEST: p. 153 #5 to 15; p. 155 #1 to 7