Date: May 7/14 **Section 4.2 – Vector Laws – Algebraically**

Recall: In **Ex. 1.** from the previous day's note we learned **geometrically** that

$\overrightarrow{AB}+\overrightarrow{BC}$

$= \overrightarrow{AC}$

Today we will learn how to add and subtract vectors **algebraically**.

Ex. 1. Add and or subtract the following vectors algebraically.

a) $\overrightarrow{PQ}+\overrightarrow{QT}$

$= \overrightarrow{PT}$

b) $\overrightarrow{ST}-\overrightarrow{UT}$

$= \overrightarrow{ST}+\overrightarrow{TU}$

$= \overrightarrow{SU}$

c) $\overrightarrow{BA}+\overrightarrow{TB}+\overrightarrow{QT}+\overrightarrow{PQ}$

$= \overrightarrow{PQ}+\overrightarrow{QT}+\overrightarrow{TB}+\overrightarrow{BA}$

$= \overrightarrow{PA}$

d) $\overrightarrow{AD}-\overrightarrow{BD}-\overrightarrow{AC}-\overrightarrow{CB}$

$= \overrightarrow{AD}+\overrightarrow{DB}+\overrightarrow{CA}+\overrightarrow{BC}$

$= \overrightarrow{AD}+\overrightarrow{DB}+\overrightarrow{BC}+\overrightarrow{CA}$

$= \overrightarrow{AA}$

$= \vec{0}$

Properties of Vector Algebra

Given $\vec{a}$, $\vec{b}$ and $\vec{c}$ are distinct non-zero vectors and k and m are real numbers, then

A. Properties of Vector Addition

*1. $\vec{a}+\vec{b} = \vec{b}+\vec{a}$

Commutative Law

*2. $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

Associative Law

3. $\vec{a}+\vec{0} = \vec{a}$

4. $\vec{a}+(-\vec{a}) = \vec{0}$

B. Properties of Scalar Multiplication

1. $(km)\vec{a} = k(m\vec{a}) = m(k\vec{a})$

Associative Law

*2. $k(\vec{a}+\vec{b}) = k\vec{a}+k\vec{b}$

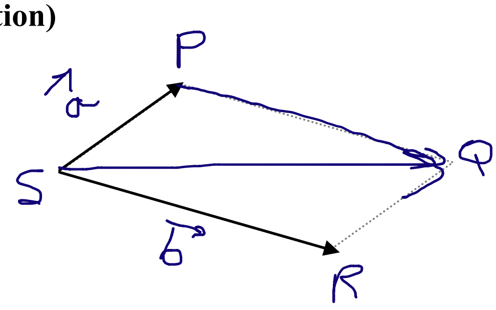
Distributive Law

3. $(k+m)\vec{a} = k\vec{a}+m\vec{a}$

Distributive Law

* We will prove these vector properties.

A. 1. Prove: $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ **(Commutative Law for Vector Addition)**



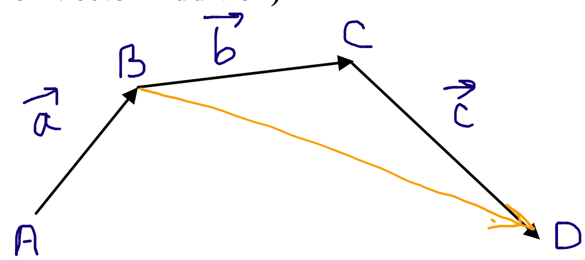
Proof: *Let $\vec{a}$ and $\vec{b}$ be distinct non-zero vectors.*

L.S.
$= \vec{a}+\vec{b}$
$= \overrightarrow{SP}+\overrightarrow{PQ}$
$= \overrightarrow{SQ}$

R.S.
$= \vec{b}+\vec{a}$
$= \overrightarrow{SR}+\overrightarrow{RQ}$
$= \overrightarrow{SQ}$

$\because$ L.S. = R.S. $\therefore \vec{a}+\vec{b}=\vec{b}+\vec{a}$

A. 2. Prove: $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ **(Associative Law for Vector Addition)**



Proof: *Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be distinct non-zero vectors.*

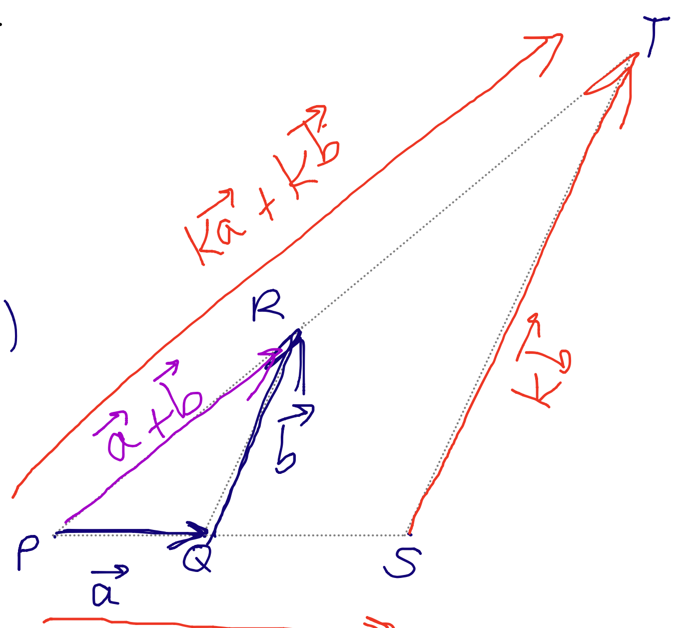
L.S.
$= (\vec{a}+\vec{b})+\vec{c}$
$= (\overrightarrow{AB}+\overrightarrow{BC})+\overrightarrow{CD}$
$= \overrightarrow{AC}+\overrightarrow{CD}$
$= \overrightarrow{AD}$

R.S.
$= \vec{a}+(\vec{b}+\vec{c})$
$= \overrightarrow{AB}+\underbrace{(\overrightarrow{BC}+\overrightarrow{CD})}$
$= \overrightarrow{AB}+\overrightarrow{BD}$
$= \overrightarrow{AD}$

$\because$ L.S. = R.S. $\therefore (\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$

B. 2. Prove: $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$ **(Distributive Law for Scalar Multiplication)**

Proof: *Let $\vec{a}$, $\vec{b}$ ~~and $\vec{c}$~~ be distinct non-zero vectors.*



Draw $\triangle PST$, where
$\overrightarrow{PS} = k\vec{a}$
$\overrightarrow{ST} = k\vec{b}$

So $\triangle PQR \sim \triangle PST$ (AA~)

$\dfrac{\overrightarrow{PS}}{\overrightarrow{PQ}} = \dfrac{\overrightarrow{ST}}{\overrightarrow{QR}} = k$

$\therefore \dfrac{\overrightarrow{PT}}{\overrightarrow{PR}} = k$

$\overrightarrow{PT} = k\overrightarrow{PR}$ $\quad k\vec{a}$

$\overrightarrow{PT} = k(\vec{a}+\vec{b})$, but $\overrightarrow{PT} = k\vec{a}+k\vec{b}$

$\therefore k(\vec{a}+\vec{b}) = k\vec{a}+k\vec{b}$

Ex. 2. Simplify.

a) $4\vec{v}+3\vec{v}-2\vec{v}$

$= 5\vec{v}$

b) $3\vec{u}+4(\vec{v}-\vec{u})$

$= 3\vec{u}+4\vec{v}-4\vec{u}$

$= 4\vec{v}-\vec{u}$

or

$-\vec{u}+4\vec{v}$

c) $4(\vec{a}+\vec{b})+3(\vec{a}-\vec{b})$

$= 4\vec{a}+4\vec{b}+3\vec{a}-3\vec{b}$

$= 7\vec{a}+\vec{b}$

d) $2(\vec{b}-\vec{a})-5(\vec{b}+\vec{a})$

$= 2\vec{b}-2\vec{a}-5\vec{b}-5\vec{a}$

$= -7\vec{a}-3\vec{b}$

or

$-3\vec{b}-7\vec{a}$

☆ test

Ex. 3. If $\vec{u}=2\vec{a}-3\vec{b}$ and $\vec{v}=4\vec{a}+5\vec{b}$ then express $\vec{a}$ and $\vec{b}$ in terms of $\vec{u}$ and $\vec{v}$.

$\vec{u}=2\vec{a}-3\vec{b}$ ①

$\vec{v}=4\vec{a}+5\vec{b}$ ②

Eliminate $\vec{b}$

① ×5 $\quad 5\vec{u}=10\vec{a}-15\vec{b}$

② ×(-3) $\quad -3\vec{v}=-12\vec{a}-15\vec{b}$

Subtract $\quad 5\vec{u}+3\vec{v}=22\vec{a}$

$$\frac{5\vec{u}}{22}+\frac{3\vec{v}}{22}=\frac{22\vec{a}}{22}$$

$$\therefore \vec{a}=\frac{5}{22}\vec{u}+\frac{3}{22}\vec{v}$$

Eliminate $\vec{a}$

① ×2 $\quad 2\vec{u}=4\vec{a}-6\vec{b}$

② $\quad \vec{v}=4\vec{a}+5\vec{b}$

Subtract $\quad 2\vec{u}-\vec{v}=-11\vec{b}$

$$\frac{2\vec{u}}{-11}-\frac{\vec{v}}{-11}=\frac{-11\vec{b}}{-11}$$

$$\therefore \vec{b}=-\frac{2}{11}\vec{u}+\frac{1}{11}\vec{v}$$

HW: pg. 133 #3, 10 to 12, 13 algebraically only

Date: May 8/14

Section 4.3 – Force as a Vector - Part I

Force - that which pulls, pushes, compresses, distorts in any way
- that which changes the state of rest or state of motion in a body
- **note:** the magnitude of a force is measured in **newtons**, **N**, where a 1 kg object weighs approximately 9.8 N

Vector Force – a vector equal in magnitude to the magnitude of a given force and having its direction parallel to the line of action of the force.

Equilibrant – the force that is equal in magnitude but opposite in direction to the resultant

*An object will be in a state of equilibrium when the **resultant** of all the forces acting on it is **zero**. This means that the three given force vectors must form a triangle or be collinear According to the **triangle inequality theorem**, a triangle can only be formed if the sum of the shorter two sides is always greater than the longest side.*

Note: Overall the sum of the smaller two magnitudes must be greater than or equal to the larger magnitude of the forces for equilibrium to be achieved.

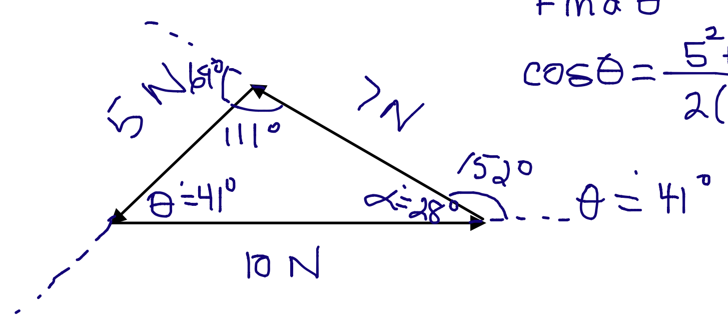
Ex. 1. Which of the following sets of forces could keep an object at rest (in a state of equilibrium)?

a) 8N, 4N and 12N — yes, $\because 8+4=12$

b) 3N, 4N and 12N — No, $\because 3+4<12$

c) 5N, 7N and 10N — yes, $\because 5+7>10$

Ex. 2. The three forces 5N, 7N and 10N are applied to an object. If the object is in a state of equilibrium, show how the forces must be arranged and calculate the angles between the lines of action of the three forces.



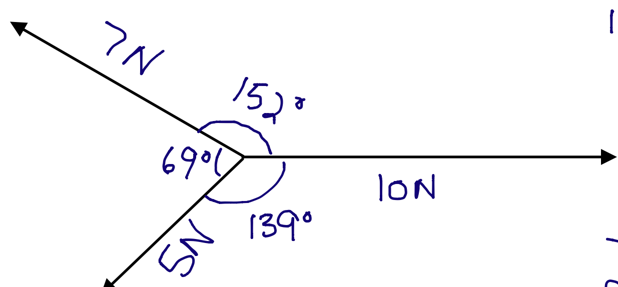
Find θ

$$\cos\theta = \frac{5^2+10^2-7^2}{2(5)(10)}$$

Find α

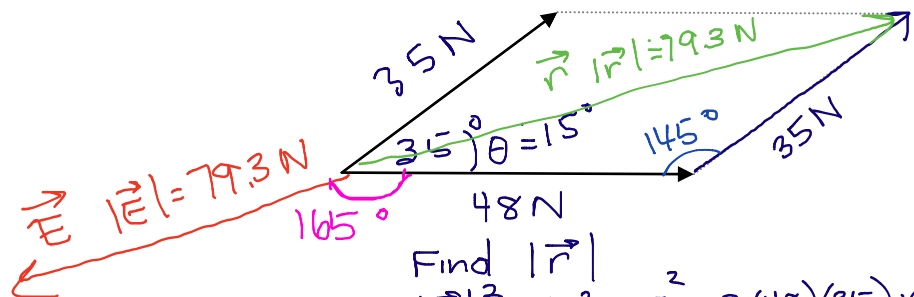
$$\cos\alpha = \frac{7^2+10^2-5^2}{2(7)(10)}$$

$\alpha \doteq 28°$



∴ the angle between the 10 N force and 5N force is about 139° and the angle between the 10N force and the 7 N force is 152° and the angle between the 7 N force and 5 N force is 69°.

Ex. 3. Find the ***resultant*** and ***equilibrant*** when forces of 48N and 35N act at an angle of 35° to each other.



Find $|\vec{r}|$

$|\vec{r}|^2 = 48^2 + 35^2 - 2(48)(35)\cdot\cos 145°$

$|\vec{r}| \doteq 79.3$

Find θ

$\cos\theta = \dfrac{48^2 + 79.3^2 - 35^2}{2(48)(79.3)}$

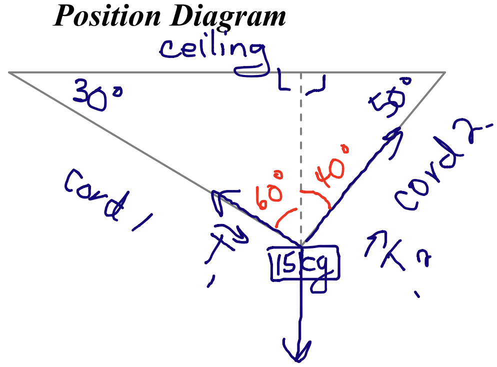
$\theta \doteq 15°$

$\therefore$ the equilibrant, $\vec{E}$, is about 79.3 N and 165° to the 48 N force.

$\therefore$ the resultant, $\vec{r}$, is about 79.3 N and 15° to the 48 N force.

Ex. 4. A 15 kg mass is suspended from the ceiling by two chords that make angles of 30° and 50° with the ceiling. Find the tensions in these chords. **Note:** A 1 kg mass exerts a force of 9.8 N.

Position Diagram



A 15 kg ~~force~~ mass exerts a force of 15×9.8 N or 147 N

Find $|\vec{T}_1|$

$\dfrac{|\vec{T}_1|}{\sin 40°} = \dfrac{147}{\sin 80°}$

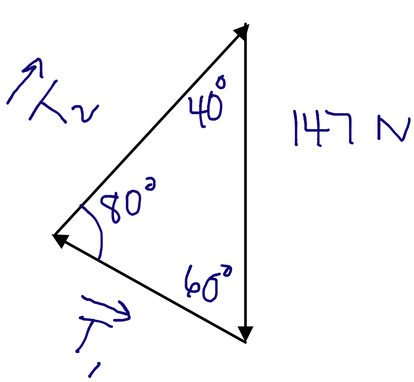
$|\vec{T}_1| \doteq 95.9$

Find $|\vec{T}_2|$

$\dfrac{|\vec{T}_2|}{\sin 60°} = \dfrac{147}{\sin 80°}$

$|\vec{T}_2| \doteq 129.3$

Vector Force Diagram



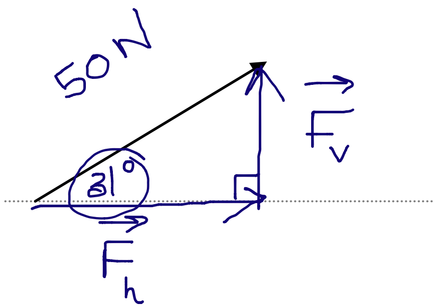
$\therefore$ the tensions in these cords are about 95.9 N and 129.3 N.

HW. pg. 141 #3, 4, 6, 7, 9 to 13

Date: May 9/14 **Section 4.3 – Force as a Vector – Part II**

Vector Components: A vector can be broken down into ***horizontal*** *and* ***vertical components.***

Ex. 1. A sled is being pulled by a force of 50N. If the rope makes an angle of 31° with the ground, find
a) the force that is pulling the sled forward (*horizontal component*)
b) the force that is pulling the sled upward (*vertical component*)



a) Find $|\vec{F}_h|$

$$\frac{|\vec{F}_h|}{50} = \cos 31^\circ$$

$$|\vec{F}_h| \doteq 42.9$$

$\therefore$ the force that is pulling the sled forward is 42.9 N

b) Find $|\vec{F}_v|$

$$\frac{|\vec{F}_v|}{50} = \sin 31^\circ$$

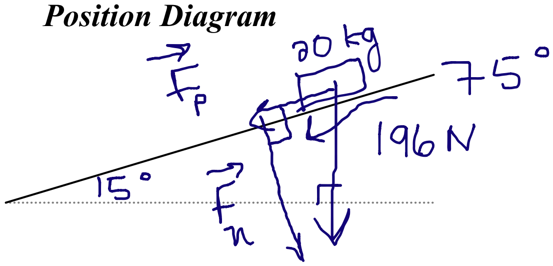
$$|\vec{F}_v| \doteq 25.8$$

$\therefore$ the force that is pulling the sled upward is 25.8 N

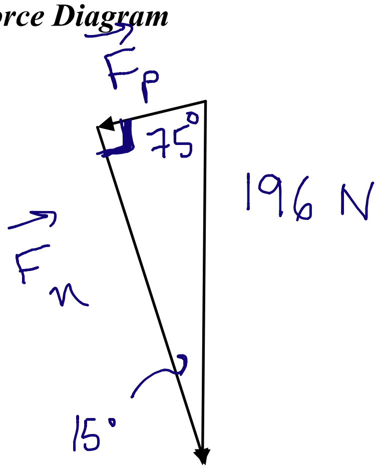
Ex. 2. A 20 kg trunk is resting on a ramp inclined at an angle of 15°. Calculate the components of the force of gravity on the trunk that are parallel and perpendicular to the ramp.

$$20\text{ kg} \times 9.8\text{ m/s}^2 = 196\text{ N}$$

Position Diagram



Vector Force Diagram



Find $|\vec{F}_p|$

$$\sin 15^\circ = \frac{|\vec{F}_p|}{196}$$

$$|\vec{F}_p| \doteq 50.7$$

Find $|\vec{F}_n|$

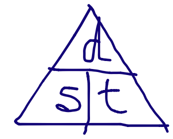
$$\cos 15^\circ = \frac{|\vec{F}_n|}{196}$$

$$|\vec{F}_n| \doteq 189.3$$

$\therefore$ the components of the force of gravity on the trunk that are parallel and perpendicular to the trunk are 50.7 N and 189.3 N respectively.

HW: pg. 141 #2, 15 to 22

Date: May 9/14



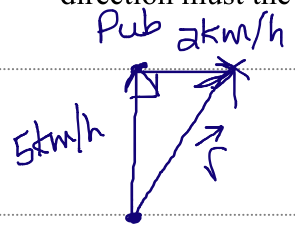
Section 4.4 – Velocity as a Vector

Ex. 1. A boat with a water speed of 5 km/h sets out to cross a 1 km wide river which flows at 2 km/h. A pub lies directly across the river on the opposite bank.

a) If the boat attempts to head straight across, how long will it take to reach the other side and how far downstream will the boat land on the opposite bank?

b) If the boat crosses the river and arrives directly at the pub on the opposite bank, in what direction must the boat steer and how long will it take to cross?

a)



$t = \frac{d}{s}$

$= \frac{1}{5}$

$\therefore$ it take $\frac{1}{5}$ h or 12 min to reach the other side.

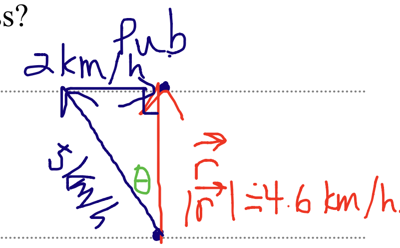
$d = s \times t$

$= 2 \times \frac{1}{5}$

$= \frac{2}{5}$

$\therefore$ he is $\frac{2}{5}$ km or 400 m down stream.

b)



Find θ

$\sin\theta = \frac{2}{5}$

$\theta = \sin^{-1}\left(\frac{2}{5}\right)$

$\theta \doteq 24°$

$\therefore$ the boat should steer 24° upstream.

Find $|\vec{r}|$

$|\vec{r}| = \sqrt{(5)^2 - (2)^2}$

$|\vec{r}| = \sqrt{21}$

$|\vec{r}| \doteq 4.6$

$t = \frac{d}{s}$

$= \frac{1}{\sqrt{21}}$

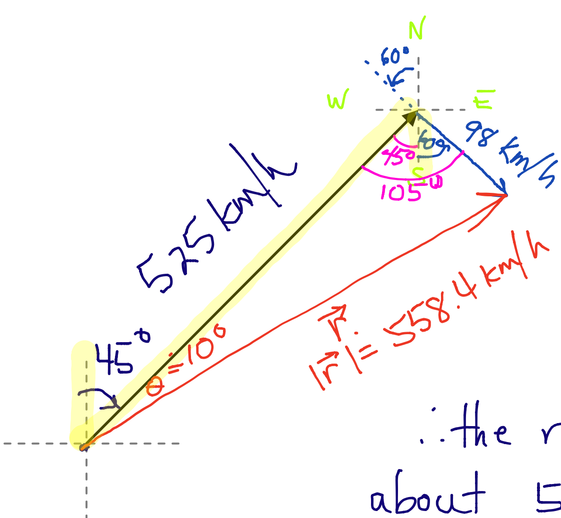
$\doteq 0.218$ h

$\therefore$ It will take about 13 min. to cross.

in still air

Ex. 2. A pilot is flying her 747 jet at 525 km/h in the direction N 45° E. She encounters a wind from N 60° W at 98 km/h. Find the resultant velocity.

ground velocity



Find $|\vec{r}|$

$|\vec{r}|^2 = 525^2 + 98^2 - 2(525)(98)\cos 105°$

$|\vec{r}| \doteq 558.4$

Find θ

$\cos\theta = \dfrac{525^2 + 558.4^2 - 98^2}{2(525)(558.4)}$

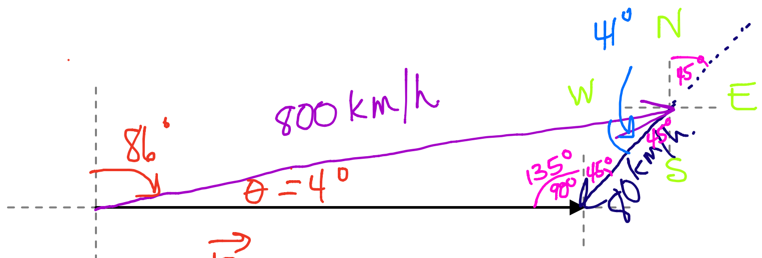
$\theta \doteq 10°$

$\therefore$ the resultant ground velocity is about 558.4 km/h, N 55° E.

Ex. 3. The pilot of an airplane that flies at 800 km/h wishes to travel to a city 800 km due east. There is an 80 km/h wind from the northeast.

a) What should the plane's heading be?

b) How long will the trip take?



$|\vec{r}| \doteq 742.2$ km/h

a) Find θ

$$\frac{\sin\theta}{80} = \frac{\sin 135°}{800}$$

$\theta \doteq 4°$

$\therefore$ the plane should head N86°E

b) Find $|\vec{r}|$

$$\frac{|\vec{r}|}{\sin 41°} = \frac{800}{\sin 135°}$$

$|\vec{r}| \doteq 742.2$

$t = \frac{d}{s}$ (h)

$= \frac{800}{742.2}$

$\doteq 1.078$

$\therefore$ the trip will take about 1 hour and 5 minutes.

HW: pg. 149 #2a, 4 to 10; Review pg. 153 #5 to 15; pg. 155 #1 to 7