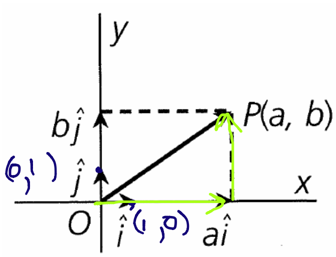
Date: May 13/14

UNIT 8 – ALGEBRAIC VECTORS AND APPLICATIONS

Section 5.1 – Coordinate Systems and Algebraic Vectors

A. Two-Dimensional Vectors $\Re^2$



Any vector in the plane can be translated so that its initial point lies at the origin.

If the coordinates of P are (a, b), then $\overrightarrow{OP} = (a, b)$ is called the position vector, and a and b are the components of the vector.

$\therefore (a, b)$ means a point or a vector.

Let $\hat{i}$ and $\hat{j}$ represent unit vectors.

$\hat{i} = (1, 0)$ and $\hat{j} = (0, 1)$ with $\vec{0} = (0, 0)$.

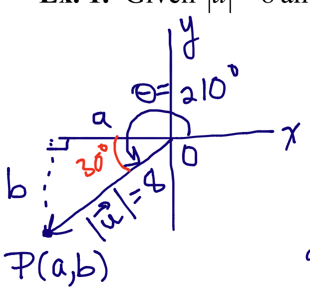
$\hat{i}$ and $\hat{j}$ are the standard basis vectors in $\Re^2$.

We can represent $\overrightarrow{OP}$ in terms of the *standard basis vectors*, where $\overrightarrow{OP} = a\hat{i} + b\hat{j}$.

Every vector in $\Re^2$ can be represented ***algebraically*** or ***geometrically***.

Algebraic Form	Geometric Form
Ordered pair notation: $\overrightarrow{OP} = (a, b)$	***Magnitude:*** $\lvert\overrightarrow{OP}\rvert$, where $\lvert\overrightarrow{OP}\rvert = \sqrt{a^2 + b^2}$
or	*and*
Unit vector notation: $\overrightarrow{OP} = a\hat{i} + b\hat{j}$	***Direction:*** θ, where θ is measured counter-clockwise from the positive x-axis to the line of the vector.
Note: $(a, b) = a\hat{i} + b\hat{j}$	

Ex. 1. Given $|\vec{u}| = 8$ and $\theta = 210^\circ$, express $\vec{u}$ as an ***algebraic vector*** in the form: **i)** (a, b) **ii)** $a\hat{i} + b\hat{j}$



Find a

$\frac{a}{8} = -\cos 30^\circ$

$\frac{a}{8} = -\frac{\sqrt{3}}{2}$

$2a = -8\sqrt{3}$

$a = -4\sqrt{3}$

Find b

$\frac{b}{8} = -\sin 30^\circ$

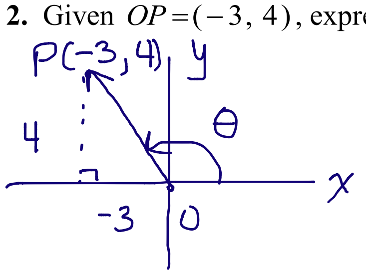
$\frac{b}{8} = -\frac{1}{2}$

$b = -4$

i) $\vec{u} = (-4\sqrt{3}, -4)$

ii) $\vec{u} = -4\sqrt{3}\,\hat{i} - 4\hat{j}$

Ex. 2. Given $\overrightarrow{OP} = (-3, 4)$, express $\overrightarrow{OP}$ as a ***geometric vector*** by stating its *magnitude* and *direction*.



r.a.a. $= \tan^{-1}\left(+\frac{4}{3}\right)$

$\doteq 53^\circ$

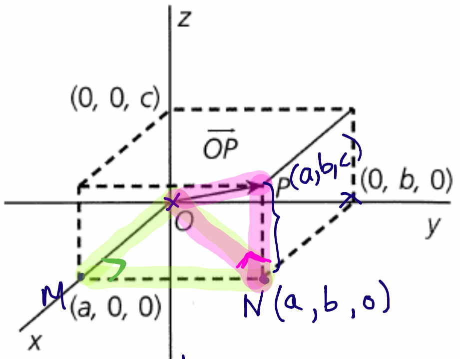
$\theta = 127^\circ$

$|\overrightarrow{OP}| = \sqrt{(-3)^2 + (4)^2}$

$|\overrightarrow{OP}| = 5$

$\therefore \theta \doteq 127^\circ$; $|\overrightarrow{OP}| = 5$

B. Three-Dimensional Vectors $\Re^3$



Any vector in the plane can be translated so that its initial point lies at the ***origin***.

If the coordinates of P are (a, b, c), then $\overrightarrow{OP} = (a, b, c)$ is called the ***position vector***, and a, b, and c are the components of the vector.

$\therefore (a, b, c)$ means a ***point*** (a, b, c) or a ***vector*** (a, b, c).

Let $\hat{i}$, $\hat{j}$ and $\hat{k}$ represent ***unit vectors*** in the positive x, y and z directions respectively.

$\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$ and $\hat{k} = (0, 0, 1)$ with $\vec{0} = (0, 0, 0)$.

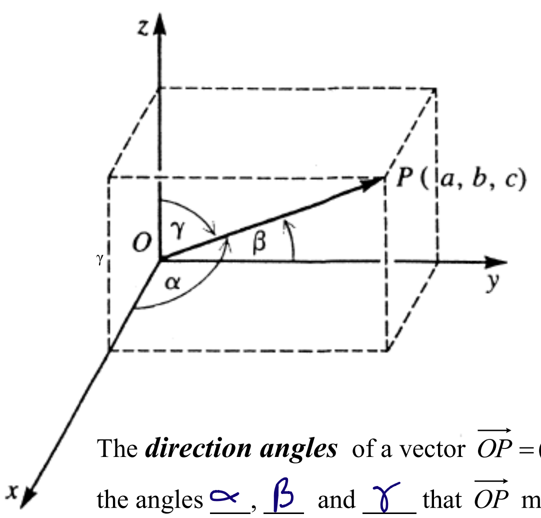
Find $|\overrightarrow{OP}|$

In $\triangle OMN$,
$|\overrightarrow{ON}|^2 = |\overrightarrow{OM}|^2 + |\overrightarrow{MN}|^2$
$|\overrightarrow{ON}|^2 = a^2 + b^2$

In $\triangle ONP$
$|\overrightarrow{OP}|^2 = |\overrightarrow{ON}|^2 + |\overrightarrow{PN}|^2$
$|\overrightarrow{OP}|^2 = a^2 + b^2 + c^2$
$\therefore |\overrightarrow{OP}| = \sqrt{a^2+b^2+c^2}$

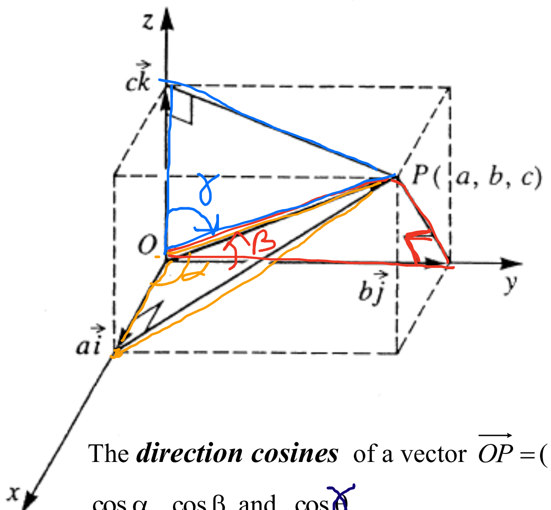
$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the ***standard basis vectors*** in $\Re^3$.

We can represent $\overrightarrow{OP}$ in terms of the ***standard basis vectors***, where $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$.
Every vector in $\Re^3$ can be represented ***algebraically*** or ***geometrically***.



The ***direction angles*** of a vector $\overrightarrow{OP} = (a, b, c)$ the angles α, β and γ that $\overrightarrow{OP}$ makes with the positive x, y and z - axes respectively.

a, b and c are called the ***direction numbers***.



The ***direction cosines*** of a vector $\overrightarrow{OP} = (a, b, c)$ are $\cos\alpha$, $\cos\beta$ and $\cos\gamma$.

$$\boxed{\cos\alpha = \frac{a}{|\overrightarrow{OP}|} \quad \cos\beta = \frac{b}{|\overrightarrow{OP}|} \quad \cos\gamma = \frac{c}{|\overrightarrow{OP}|}}$$

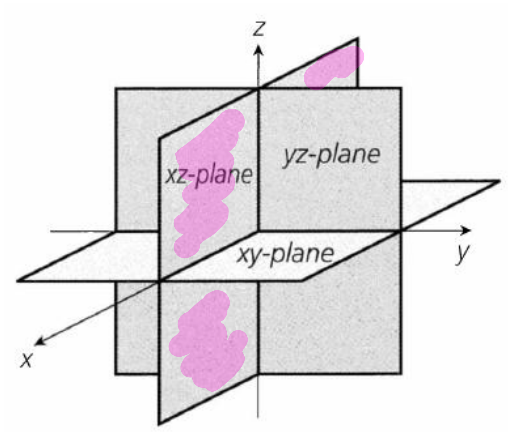
Every vector in $\Re^2$ can be represented ***algebraically*** or ***geometrically***.

Algebraic Form	Geometric Form
Ordered triple notation: $\overrightarrow{OP} = (a, b, c)$	***Magnitude:*** $\lvert\overrightarrow{OP}\rvert$, where $\lvert\overrightarrow{OP}\rvert = \sqrt{a^2+b^2+c^2}$
or	*and*
Unit vector notation: $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$	***Direction angles:*** α, β and γ that $\overrightarrow{OP}$ makes with the positive x, y and z - axes respectively.
Note: $(a, b, c) = a\hat{i} + b\hat{j} + c\hat{k}$	

Note: A ***unit vector*** in the direction of $\overrightarrow{OP}$ is $\hat{r} = (\cos\alpha, \cos\beta, \cos\gamma)$ $\because$ a unit vector has a length of 1,

$$\widehat{OP} = \frac{1}{|\overrightarrow{OP}|}\overrightarrow{OP}$$
$$= \frac{1}{\sqrt{a^2+b^2+c^2}}(a, b, c)$$
$$= \left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$$

$$\sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma} = 1$$
square both sides
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



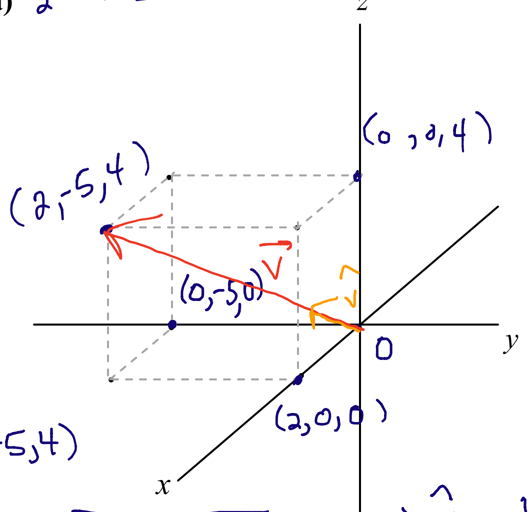
A ***plane*** in space that contains ***two*** of the coordinate axes is known as a ***coordinate plane***. The plane containing the *x*- and *y*- axes, for instance, is called the *xy*-plane. The other two coordinate planes are named similarly. A point such as $(-4, 0, 1)$, which has a *y*-coordinate of 0, lies in the *xz*-plane.

To plot a point $P\ (a, b, c)$ in space, move *a* units from the origin in the *x* direction, *b* units in the *y* direction, and then *c* units in the *z* direction. Be sure each move is made along a line parallel to the corresponding axis. Drawing a rectangular box will help you to see the three-dimensional aspect of such diagrams.

Ex. 3. Given vector $\vec{v} = (2, -5, 4)$,

a) graph $\vec{v}$.
b) find the magnitude of $\vec{v}$.
c) find the direction cosines.
d) find the direction angles.
e) find $\hat{v}$

a) $\frac{1}{2}$ cm = 1 unit



$\vec{v} = (2, -5, 4)$

b) $|\vec{v}| = \sqrt{a^2+b^2+c^2}$
$= \sqrt{(2)^2+(-5)^2+(4)^2}$
$= \sqrt{45}$
$\therefore |\vec{v}| = 3\sqrt{5}$

c) $\cos\alpha = \frac{a}{|\vec{v}|}$ $\quad \cos\beta = \frac{b}{|\vec{v}|}$ $\quad \cos\gamma = \frac{c}{|\vec{v}|}$

$\cos\alpha = \frac{2}{3\sqrt{5}}$, $\quad \cos\beta = \frac{-5}{3\sqrt{5}}$ $\quad \cos\gamma = \frac{4}{3\sqrt{5}}$

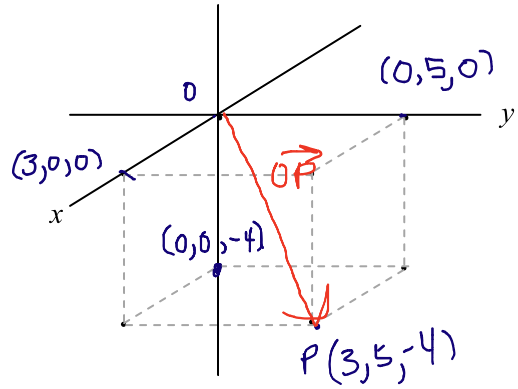
d) $\alpha \doteq 73°$ $\quad \beta \doteq 138°$ $\quad \gamma \doteq 53°$

e) $\hat{v} = \frac{1}{|\vec{v}|}\vec{v}$
$= \frac{1}{3\sqrt{5}}(2, -5, 4)$
$\therefore \hat{v} = \left(\frac{2}{3\sqrt{5}}, \frac{-5}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}\right)$

Ex. 4. Given vector $\overrightarrow{OP} = 3\hat{i} + 5\hat{j} - 4\hat{k}$,

a) graph $\overrightarrow{OP}$. $\quad \overrightarrow{OP} = (3, 5, -4)$
b) find the magnitude of $\overrightarrow{OP}$.
c) find the direction cosines.
d) find the direction angles.
e) find a unit vector in the direction opposite to $\overrightarrow{OP}$.

a)



b) $|\overrightarrow{OP}| = 5\sqrt{2}$

c) $\cos\alpha = \frac{3}{5\sqrt{2}}$, $\cos\beta = \frac{\cancel{5}}{\cancel{5}\sqrt{2}}$, $\cos\gamma = \frac{-4}{5\sqrt{2}}$
$= \frac{1}{\sqrt{2}}$

d) $\alpha \doteq 65°$, $\beta \doteq 45°$, $\gamma \doteq 124°$

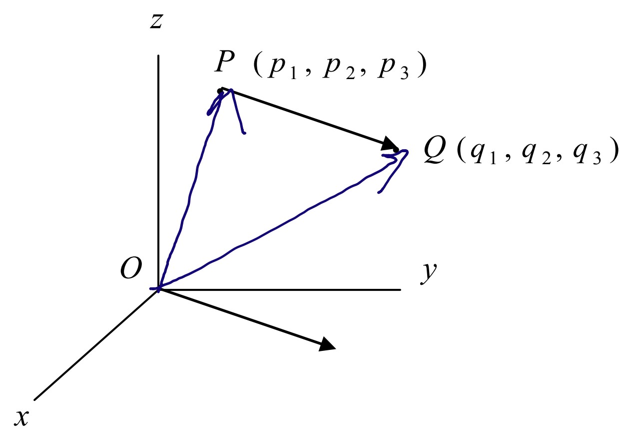
e) $-\widehat{OP} = \left(-\frac{3}{5\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{4}{5\sqrt{2}}\right)$

H.W. pg. 166 #1 to 5, 6ab, 7bc, 8 to 12, 13acef, 14, 16 to 20, 22, 23 & Worksheet #1 to 4

Date: May 15/14

Section 5.2 – Operations With Algebraic Vectors

I The Vector Joining Two Points



$\overrightarrow{PQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$

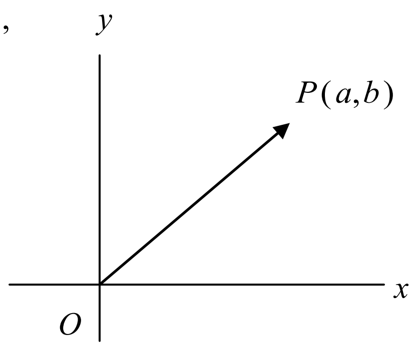
$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\overrightarrow{PQ} = (q_1, q_2, q_3) - (p_1, p_2, p_3)$

$\overrightarrow{PQ} = (q_1 - p_1, q_2 - p_2, q_3 - p_3)$

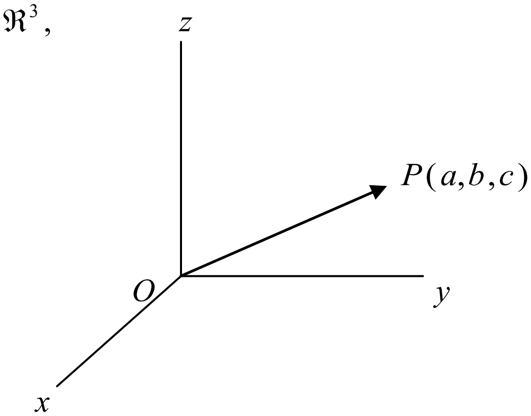
II The Magnitude of a Vector

In $\Re^2$,



$$|\overrightarrow{OP}| = \sqrt{a^2 + b^2}$$

In $\Re^3$,



$$|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$$

Ex. 1. Given the points $A(1,1,2)$, $B(2,-1,3)$ and $C(4,1,5)$, find:

a) $\overrightarrow{OA} + \overrightarrow{OB}$

$= (1,1,2) + (2,-1,3)$

$= (3,0,5)$

b) $2\overrightarrow{OB} - 3\overrightarrow{OC}$

$= 2(2,-1,3) - 3(4,1,5)$

$= (4-12, -2-3, 6-15)$

$= (-8,-5,-9)$

c) $\overrightarrow{BC}$

$= \overrightarrow{OC} - \overrightarrow{OB}$

$= (4,1,5) - (2,-1,3)$

$= (2,2,2)$

d) $\overrightarrow{CA}$

$= \overrightarrow{OA} - \overrightarrow{OC}$

$= (1,1,2) - (4,1,5)$

$= (-3,0,-3)$

Ex. 2. If $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$, find $|\vec{a} - \vec{b}|$.

$\vec{a} = (2,-3,4)$; $\vec{b} = (3,1,-1)$

$\vec{a} - \vec{b} = (2,-3,4) - (3,1,-1)$

$\therefore \vec{a} - \vec{b} = (-1,-4,5)$

$|\vec{a} - \vec{b}| = \sqrt{(-1)^2 + (-4)^2 + (5)^2}$

$= \sqrt{42}$

$\therefore |\vec{a} - \vec{b}| = \sqrt{42}$ units.

Ex. 3. Given the points $P(4,3,5)$ and $Q(1,-2,5)$, find $|\overrightarrow{PQ}|$.

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (1,-2,5) - (4,3,5)$

$\therefore \overrightarrow{PQ} = (-3,-5,0)$

$|\overrightarrow{PQ}| = \sqrt{(-3)^2 + (-5)^2 + (0)^2}$

$= \sqrt{34}$

$\therefore |\overrightarrow{PQ}| = \sqrt{34}$ units

Ex. 4. Given the points $A(5,-1)$, $B(-3,4)$ and $C(13,-6)$, show that A, B and C are ***collinear using vectors***.

Note: A, B and C are ***collinear*** if $\underline{\overrightarrow{AC}}$ is a ***scalar multiple*** of $\underline{\overrightarrow{AB}}$.

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (13,-6) - (5,-1)$

$\therefore \overrightarrow{AC} = (8,-5)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

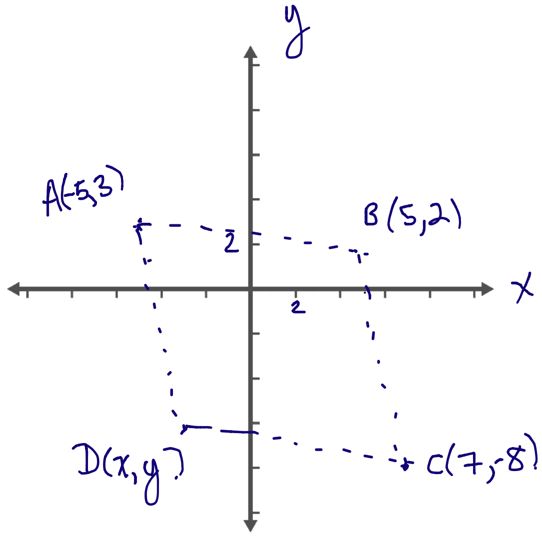
$= (-3,4) - (5,-1)$

$\therefore \overrightarrow{AB} = (-8,5)$

$\because \overrightarrow{AC} = k\overrightarrow{AB}$ where $k = -1$

$\therefore$ the points A, B & C are collinear.

Ex. 5. If quadrilateral *ABCD* is a parallelogram with vertices $A(-5,3)$, $B(5,2)$ and $C(7,-8)$, find the coordinates of *D*, ***using vectors***.



Let $D(x,y)$ be the fourth vertex.

$\overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OD}$

$(5,2)-(-5,3) = (7,-8)-(x,y)$

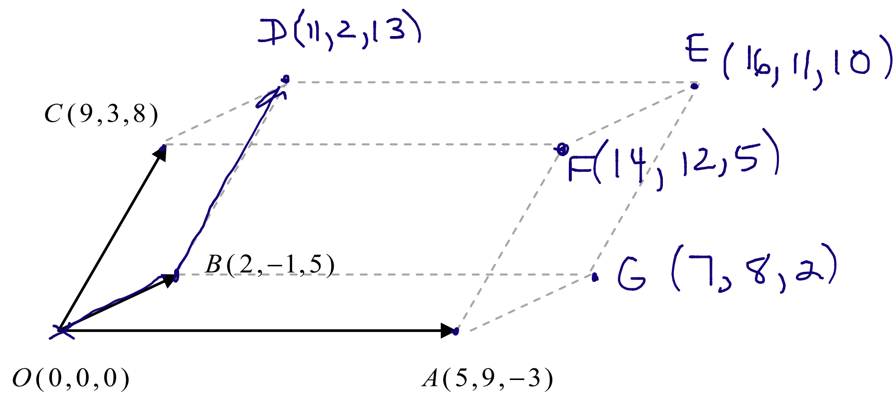
$(10,-1) = (7-x, -8-y)$

$\therefore 10 = 7-x$ & $-1 = -8-y$

$x = -3$ $\quad y = -7$

$\therefore$ the coordinate of D are $(-3,-7)$.

Ex. 6. If $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are three edges of a ***parallelepiped*** where O is $(0,0,0)$, A is $(5,9,-3)$, B is $(2,-1,5)$ and C is $(9,3,8)$. Find the coordinates of the other four vertices, *D*, *E*, *F* and *G*.



$\therefore$ the other four vertices are $D(11,2,13)$, $E(16,11,10)$, $F(14,12,5)$ and $G(7,8,2)$

Find D

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= \overrightarrow{OB} + \overrightarrow{OC}$

$= (2,-1,5) + (9,3,8)$

$= (11,2,13)$

Find E

$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BD} + \overrightarrow{DE}$

$= \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OA}$

$= (2,-1,5) + (9,3,8) + (5,9,-3)$

$= (16,11,10)$

Find F

$\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OC}$

$= (5,9,-3) + (9,3,8)$

$= (14,12,5)$

Find G

$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OB}$

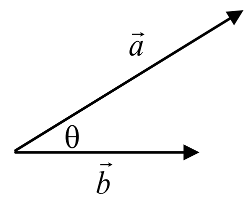
$= (5,9,-3) + (2,-1,5)$

$= (7,8,2)$

HW. pg. 172 #2agij, 3ab, 4f, 5d, 6c, 8, 9c, 12, 13ad, 14ab, 15, 16

Date: May 16/14 **Section 5.3 – The Dot Product of Two Vectors**

A. The Dot Product in Vector Form



The ***dot product*** of any two vectors $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

where θ is the angle between the vectors.

Note: The ***dot product*** of two vectors is a ***scalar***.

Ex. 1. Complete the following.

a) If the angle, θ, between the vectors is ***acute*** then $0^\circ < \theta < 90^\circ$ and $\vec{a}\cdot\vec{b} > 0$.

b) If the angle, θ, between the vectors is ***obtuse*** then $90^\circ < \theta < 180^\circ$ and $\vec{a}\cdot\vec{b} < 0$.

c) If the angle, θ, between the vectors is ***right*** then $\theta = 90^\circ$ and $\vec{a}\cdot\vec{b} = 0$.

Ex. 2. If $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $\theta = 60^\circ$, then find $\vec{a}\cdot\vec{b}$.

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= (5)(6)\cos 60^\circ$

$= 30\left(\frac{1}{2}\right)$

$= 15$

$\therefore \vec{a}\cdot\vec{b} = 15$ scalar

Ex. 3. If $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, determine each of the following:

a) $\hat{i}\cdot\hat{i}$

$= (1)(1)\cos 0^\circ$

$= (1)(1)(1)$

$= 1$

d) $\hat{i}\cdot\hat{j}$

$= (1)(1)\cos 90^\circ$

$= (1)(1)(0)$

$= 0$

b) $\hat{j}\cdot\hat{j}$

$= 1$

e) $\hat{i}\cdot\hat{k}$

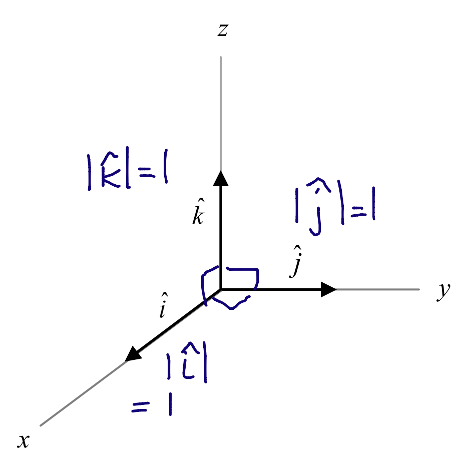
$= 0$

c) $\hat{k}\cdot\hat{k}$

$= 1$

f) $\hat{j}\cdot\hat{k}$

$= 0$



B. The Dot Product in Component Form

Let $\vec{a}=(a_1,a_2,a_3)$ and $\vec{b}=(b_1,b_2,b_3)$

$\vec{a}\cdot\vec{b}$

$=(a_1,a_2,a_3)\cdot(b_1,b_2,b_3)$

$=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})\cdot(b_1\hat{i}+b_2\hat{j}+b_3\hat{k})$

$=a_1b_1(\hat{i}\cdot\hat{i})+a_1b_2(\hat{i}\cdot\hat{j})+a_1b_3(\hat{i}\cdot\hat{k})+a_2b_1(\hat{j}\cdot\hat{i})+a_2b_2(\hat{j}\cdot\hat{j})+a_2b_3(\hat{j}\cdot\hat{k})+a_3b_1(\hat{k}\cdot\hat{i})+a_3b_2(\hat{k}\cdot\hat{j})+a_3b_3(\hat{k}\cdot\hat{k})$

$=a_1b_1(1)+a_1b_2(0)+a_1b_3(0)+a_2b_1(0)+a_2b_2(1)+a_2b_3(0)+a_3b_1(0)+a_3b_2(0)+a_3b_3(1)$

$=a_1b_1+a_2b_2+a_3b_3$

the ***dot product*** of any two vectors $\vec{a}$ and $\vec{b}$ in component form is

$$\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$$

Ex. 4. Find $\vec{a}\cdot\vec{b}$ if $\vec{a}=(1,2,-3)$ and $\vec{b}=2\hat{i}-3\hat{j}+\hat{k}$.

$\vec{a}\cdot\vec{b}=(1,2,-3)\cdot(2,-3,1)$

$=(1)(2)+(2)(-3)+(-3)(1)$

$=2-6-3$

$=-7$

$\therefore\ \vec{a}\cdot\vec{b}=-7$

Ex. 5. Determine whether or not $\vec{u}=(1,2,3)$ and $\vec{v}=(3,-4,-2)$ are ***perpendicular***.

$\vec{u}\cdot\vec{v}=(1,2,3)\cdot(3,-4,-2)$

$=3-8-6$

$=-11$

$\because\ \vec{u}\cdot\vec{v}\neq 0$, $\therefore$ $\vec{u}$ and $\vec{v}$ are not perpendicular.

Ex. 6. Find the angle θ between the vectors $\vec{a}=(2,-1,4)$ and $\vec{b}=(-3,1,2)$.

$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta=\dfrac{1}{\sqrt{21}\cdot\sqrt{14}}$

$\theta\doteq 87°$

$\vec{a}\cdot\vec{b}=-6-1+8$

$=1$

$|\vec{a}|=\sqrt{4+1+16}$

$=\sqrt{21}$

$|\vec{b}|=\sqrt{9+1+4}$

$=\sqrt{14}$

$\therefore$ the angle between the vector is about $87°$.

Ex. 7. For what values of p will the vectors of $\vec{a}=(1,p,2)$ and $\vec{b}=(3,-9,6)$ be

i) ***collinear***?

$\vec{b}=k\vec{a}$

$(3,-9,6)=3(1,p,2)$

$(3,-9,6)=(3,3p,6)$

$\therefore\ -9=3p$

$\therefore\ p=-3$

ii) ***perpendicular***?

$\vec{a}\cdot\vec{b}=0$

$(1,p,2)\cdot(3,-9,6)=0$

$3-9p+12=0$

$15-9p=0$

$-9p=-15$

$\therefore\ p=\dfrac{5}{3}$

Ex. 8. Find a vector ***perpendicular*** to

i) $(5,-2)$

$(5,-2)\cdot(2,5)=0$

or $(5,-2)\cdot(-4,-10)=0$

ii) $(4,-1,2)$

$(4,-1,2)\cdot(1,2,-1)=0$

or $(4,-1,2)\cdot(4,2,-7)=0$

C. Properties of the Dot Product

1. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ ***Commutative Law***

***2.** $\vec{a}\cdot\vec{a}=|\vec{a}|^2$

3.** $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ ***Distributive Law

4. $(k\vec{a})\cdot\vec{b}=\vec{a}\cdot(k\vec{b})=k(\vec{a}\cdot\vec{b})$ ***Associative Law***

* We will prove these properties of the dot product by example.

Ex. 9. If $\vec{a}=(-2,3,1)$, $\vec{b}=(5,6,-7)$ and $\vec{c}=(3,-2,4)$, verify that

a) $\vec{a}\cdot\vec{a}=|\vec{a}|^2$

L.S.

$= \vec{a}\cdot\vec{a}$

$=(-2,3,1)\cdot(-2,3,1)$

$= 4+9+1$

$= 14$

R.S.

$= |\vec{a}|^2$

$= \left(\sqrt{(-2)^2+(3)^2+(1)^2}\right)^2$

$= 4+9+1$

$= 14$

$\because$ L.S. = R.S

$\therefore \vec{a}\cdot\vec{a}=|\vec{a}|^2$

b) $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

L.S.

$=\vec{a}\cdot(\vec{b}+\vec{c})$

$=(-2,3,1)\cdot[(5,6,-7)+(3,-2,4)]$

$=(-2,3,1)\cdot(8,4,-3)$

$=-16+12-3$

$=-7$

R.S.

$=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

$=[(-2,3,1)\cdot(5,6,-7)]+[(-2,3,1)\cdot(3,-2,4)]$

$=(-10+18-7)+(-6-6+4)$

$=(1)+(-8)$

$=-7$

$\because$ L.S. = R.

$\therefore \vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$.

Ex. 10. If $\vec{a}$ and $\vec{b}$ are distinct ***unit vectors*** and the angle between them is 120°, calculate $(2\vec{a}+3\vec{b})\cdot(4\vec{a}-5\vec{b})$.

$= 8(\vec{a}\cdot\vec{a})+2(\vec{a}\cdot\vec{b})-15(\vec{b}\cdot\vec{b})$

$= 8|\vec{a}|^2+2\underbrace{(\vec{a}\cdot\vec{b})}-15|\vec{b}|^2$

$= 8|\vec{a}|^2+2|\vec{a}||\vec{b}|\cos\theta-15|\vec{b}|^2$

sub in $|\vec{a}|=1$, $|\vec{b}|=1$ and $\theta=120^\circ$

$= 8(1)^2+2(1)(1)\cdot\cos 120^\circ-15(1)^2$

$= 8+2\left(-\frac{1}{2}\right)-15$

$= 8-1-15$

$= -8$

$\therefore (2\vec{a}+3\vec{b})\cdot(4\vec{a}-5\vec{b})=-8$

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