Date:_____ **UNIT 1: ALGEBRAIC & NUMERICAL EXPRESSIONS**

1.1 Adding, Subtracting, and Multiplying Polynomials

1. Simplify.

a) $(3x^2+7x+1)+(2x^2-3x-5)$

$= 3x^2+7x+1+2x^2-3x-5$

$= 5x^2+4x-4$

b) $(5x^2-2x-3)-(3x^2+6x-1)$

$= 5x^2-2x-3-3x^2-6x+1$

$= 2x^2-8x-2$

c) $\sqrt{25x^6}-6y^2+\sqrt{x^2y^2}-7x^3-\sqrt{9y^4}$

$= 5x^3-6y^2+xy-7x^3-3y^2$

$= -2x^3+xy-9y^2$

d) $\sqrt{49(x-2)^2}-\sqrt{(3x+1)^2}$

$= 7(x-2)-(3x+1)$

$= 7x-14-3x-1$

$= 4x-15$

e) $2x^2(x+y)-3x^2(x+4y)$

$= 2x^3+2x^2y-3x^3-12x^2y$

$= -x^3-10x^2y$

f) $2[2(4x+3)-2(x-1)]$

$= 2(8x+6-2x+2)$

$= 2(6x+8)$

$= 12x+16$

2. Expand and simplify.

a) $(3x^2-1)(2x^2+5)$

$= 6x^4+15x^2-2x^2-5$

$= 6x^4+13x^2-5$

b) $-2(x+3y)(2x-y)$

$= -2(2x^2-xy+6xy-3y^2)$

$= -2(2x^2+5xy-3y^2)$

$= -4x^2-10xy+6y^2$

c) $3(2x-1)^2-(3x-4)(x+1)-2(3x-1)(3x+1)$

$= 3[(2x-1)(2x-1)]-[(3x-4)(x+1)]-2[(3x-1)(3x+1)]$

$= 3(4x^2-2x-2x+1)-(3x^2+3x-4x-4)-2(9x^2+3x-3x-1)$

$= 3(4x^2-4x+1)-(3x^2-x-4)-2(9x^2-1)$

$= 12x^2-12x+3-3x^2+x+4-18x^2+2$

$= -9x^2-11x+9$

d) $(x^2-2x+4)(x+2)^2$

$= (x^2-2x+4)[(x+2)(x+2)]$

$= (x^2-2x+4)(x^2+2x+2x+4)$

$= (x^2-2x+4)(x^2+4x+4)$

$= x^4+4x^3+4x^2-2x^3-8x^2-8x+4x^2+16x+16$

$= x^4+2x^3+8x+16.$

HW. Exercise 1.1

Date:_____ **1.2 Review of Factoring Techniques**

"When factoring always look for the *greatest common factor* first."

Common Factoring

1. Factor.

a) $-9x+6xy$

$=-3x(3-2y)$

b) $8x^2y^3-6x^3y^3+2x^2y$

$=2x^2y(4y^2-3xy^2+1)$

Difference of Squares $a^2-b^2=(a-b)(a+b)$ or $a^2-b^2=\left(\sqrt{a^2}-\sqrt{b^2}\right)\left(\sqrt{a^2}+\sqrt{b^2}\right)$

2. Factor completely.

a) $4x^2-1$

$=(2x-1)(2x+1)$

b) $36x^2-25y^4$

$=(6x-5y^2)(6x+5y^2)$

c) $81-a^4$

$=(9-a^2)(9+a^2)$

$=(3-a)(3+a)(9+a^2)$

or $=-(a-3)(a+3)(a^2+9)$

d) $-3a^7b^4+48a$

$=-3a(a^6b^4-16)$

$=-3a(a^3b^2-4)(a^3b^2+4)$

Simple Trinomials $ax^2+bx+c,\ a=1$ ***or*** $ax^4+bx^2+c,\ a=1$

3. Factor completely.

a) $x^2+8x+12$

$=(x+6)(x+2)$

b) $x^2-3xy-28y^2$

$=(x-7y)(x+4y)$

c) $-2y^3-14y^2+36y$

$=-2y(y^2+7y-18)$

$=-2y(y+9)(y-2)$

d) x^4-10x^2+9

$=(x^2-9)(x^2-1)$

$=(x-3)(x+3)(x-1)(x+1)$

Tricky Trinomials $ax^2+bx+c,\ a\neq 1$ ***or*** $ax^4+bx^2+c,\ a\neq 1$

4. Factor completely.

a) $3x^2+x-10$

x #

$3x^{-5}$

1^{+2}

$=(3x-5)(x+2)$

b) $5x^2-11x-12$

x #

$5x^{+4}$

1^{-3}

$=(5x+4)(x-3)$

c) $4a^2-20ab+25b^2$

a b

2^{-5}

2^{-5}

$=(2a-5b)(2a-5b)$

$=(2a-5b)^2$

d) $4y^2+23y+15$

y #

4^{3}

1^{5}

$=(4y+3)(y+5)$

e) $12x^4+34x^3-28x^2$

$=2x^2(6x^2+17x-14)$

$=2x^2(3x-2)(2x+7)$

x #

3^{-2}

2^{7}

f) $2u^4-13u^2+20$

u^2 #

2^{-5}

1^{-4}

$=(2u^2-5)(u^2-4)$

$=(2u^2-5)(u-2)(u+2)$

HW. Exercise 1.2

Date:_____ **1.3 Factoring by Substitution and Grouping**

Warm-up:

Factor completely. ***(Remember to always look for a greatest common factor first.)***

a) $-9x^2+18x-9$

$=-9(x^2-2x+1)$
$=-9(x-1)(x-1)$
$=-9(x-1)^2$

b) x^4-3x^2-4

$=(x^2-4)(x^2+1)$
$=(x-2)(x+2)(x^2+1)$

c) $4x^2+15xy-4y^2$

$=(4x-y)(x+4y)$

x y
4 −1
1 4

d) $12m^3-46m^2-36m$

$=2m(6m^2-23m-18)$
$=2m(3m+2)(2m-9)$

m #
3 2
2 −9

Factoring by Substiution

1. Factor completely using the method of ***substitution***.

a) $6x(x+1)-2(x+1)$

Let $y=x+1$

$=6xy-2y$
$=2y(3x-1)$
$=2(x+1)(3x-1)$

b) $(x+2)^2-6(x+2)+8$

Let $y=x+2$

$=y^2-6y+8$
$=(y-4)(y-2)$
$=[(x+2)-4][(x+2)-2]$
$=x(x-2)$

c) $(x^2+2x)^2-2(x^2+2x)-3$

Let $y=x^2+2x$

$=y^2-2y-3$
$=(y-3)(y+1)$
$=[(x^2+2x)-3][(x^2+2x)+1]$
$=(x^2+2x-3)(x^2+2x+1)$
$=(x+3)(x-1)(x+1)(x+1)$
$=(x+3)(x-1)(x+1)^2$

h) $9(3x+1)^2-(x-5)^2$

Let $a=3x+1$
$b=x-5$

$=9a^2-b^2$
$=(3a-b)(3a+b)$
$=[3(3x+1)-(x-5)][3(3x+1)+(x-5)]$
$=(9x+3-x+5)(9x+3+x-5)$
$=(8x+8)(10x-2)$
$=8(x+1)(2)(5x-1)$
$=16(x+1)(5x-1)$

Factoring by Grouping

2. Factor the following by ***grouping***.

a) $5x^3-10x^2-3x+6$

$=5x^2(x-2)-3(x-2)$
$=(x-2)(5x^2-3)$

b) $2x^3+8x^2+x+4$

$=2x^2(x+4)+1(x+4)$
$=(x+4)(2x^2+1)$

c) $x^4+6x^2+9-4y^4$

$=(x^2+3)^2-4y^4$
$=[(x^2+3)-2y^2][(x^2+3)+2y^2]$
$=(x^2-2y^2+3)(x^2+2y^2+3)$

d) $x^2-y^2+14y-49$

$=x^2-(y^2-14y+49)$
$=x^2-(y-7)^2$
$=[x-(y-7)][x+(y-7)]$
$=(x-y+7)(x+y-7)$

HW. Exercise 1.3

Date:_____

1.4 Solving Quadratic Equations by Factoring

Recall: *Every quadratic equation of the form* $ax^2+bx+c=0$ *is of the* 2^{nd} *degree and has 2 roots.*

1. Solve each of the following for x by factoring.

a) $9x^2-25y^2=0$

$(3x-5y)(3x+5y)=0$

$\therefore 3x-5y=0$ or $3x+5y=0$

$\frac{3x}{3}=\frac{5y}{3}$ $\quad 3x=-5y$

$x=\frac{5}{3}y$ $\quad x=-\frac{5}{3}y$

b) $-3x+6x^2=0$

$-3x(1-2x)=0$

$\therefore -3x=0$ or $1-2x=0$

$x=0$ $\quad -2x=-1$

$x=\frac{1}{2}$

c) $x^2-4x=12$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$\therefore x-6=0$ or $x+2=0$

$x=6$ $\quad x=-2$

d) $2x^2=15-7x$

$2x^2+7x-15=0$

$(2x-3)(x+5)=0$

$\therefore 2x-3=0$ or $x+5=0$

$2x=3$ $\quad x=-5$

$x=\frac{3}{2}$

e) $-9x^2=16+24x$

$0=9x^2+24x+16$

$0=(3x+4)(3x+4)$

$\therefore 3x+4=0$ or $3x+4=0$

$3x=-4$ or $x=-\frac{4}{3}$

$x=-\frac{4}{3}$

f) $18xy-18y^2+8x^2=0$

$8x^2+18xy-18y^2=0$

$2(4x^2+9xy-9y^2)=0$

$2(4x-3y)(x+3y)=0$

$\therefore 4x-3y=0$ or $x+3y=0$

$4x=3y$ $\quad x=-3y$

$x=\frac{3y}{4}$

2. Solve.

a) $3(a-2)(a+2)+6=2(a-3)$

$3(a^2-4)+6=2(a-3)$

$3a^2-12+6=2a-6$

$3a^2-6=2a-6$

$3a^2-2a=0$

$a(3a-2)=0$

$\therefore a=0$ or $3a-2=0$

$3a=2$

$a=\frac{2}{3}$

b) $2t(3-t)=3t+7-4(t-1)^2$

$2t(3-t)=3t+7-4(t-1)(t-1)$

$2t(3-t)=3t+7-4(t^2-2t+1)$

$6t-2t^2=3t+7-4t^2+8t-4$

$6t-2t^2=-4t^2+11t+3$

$2t^2-5t-3=0$

$(2t+1)(t-3)=$

$\therefore 2t+1=0$ or $t-3=0$

$t=-\frac{1}{2}$ $\quad t=3$

HW. Exercise 1.4
Unit 1 Part I Test covers Days 1 to 4
HW. Part I Review 1.1 to 1.4