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## 1.5 Simplifying Rational Expressions

**Definition:** A rational expression is of the form  $\frac{f}{g}$ , where  $f$  and  $g$  are polynomials and  $g \neq 0$ .

- To simplify rational expressions:**
- i) factor the numerators and denominators fully
  - ii) state any restrictions on the variables in the denominators
  - iii) reduce

1. Simplify each of the following. State any restrictions on the variables.

a)  $\frac{-3a^3b^2c}{-9ab^5c^2}$  If  $-9ab^5c^2 \neq 0$   
 then:  
 $a \neq 0$   
 $b \neq 0$   
 $c \neq 0$

$$= \frac{-\cancel{3}a^{\cancel{3}}\cancel{b}^2\cancel{c}}{\cancel{-9}^3\cancel{a}^1\cancel{b}^5\cancel{c}^2} = \frac{-a^2}{3b^3c}$$

b)  $\frac{x(x-5)(2x+4)}{(x^2+2x)(2x-3)}$  If  $x(x+2)(2x-3) \neq 0$   
 then:  $x \neq 0$   
 $x \neq -2$   
 $x \neq \frac{3}{2}$

$$= \frac{\cancel{x}(x-5)\cancel{(2x+4)}^2}{\cancel{x}(x+2)\cancel{(2x-3)}} = \frac{2(x-5)}{2x-3}$$

2. Simplify each of the following. State any restrictions on the variables.

a)  $\frac{20x^3 + 5x^2 - 10x}{5x}$

$$= \frac{\cancel{5x}(4x^2 + x - 2)}{\cancel{5x}^1} = 4x^2 + x - 2, \quad x \neq 0$$

b)  $\frac{4x^2 - 4x - 15}{4x^2 + 16x + 15}$

$$= \frac{\cancel{(2x+5)}(2x-5)}{\cancel{(2x+5)}(2x+3)} = \frac{2x-5}{2x+5}, \quad x \neq -\frac{5}{2}, -\frac{3}{2}$$

c)  $\frac{3y-2x}{4x-6y}$

$$= \frac{-2x+3y}{4x-6y} = \frac{-(2x-3y)}{2(2x-3y)}$$

If  $2x-3y \neq 0$   
 then:  $x \neq \frac{3}{2}y$

$$= -\frac{1}{2}$$

d)  $\frac{9y^2 - 6y + 1}{9y^2 - 21y + 6}$

$$= \frac{(3y-1)(3y-1)}{3(3y^2-7y+2)} = \frac{\cancel{(3y-1)}(3y-1)}{3\cancel{(3y-1)}(y-2)}$$

$$= \frac{3y-1}{3(y-2)}, \quad y \neq \frac{1}{3}, 2$$

e)  $\frac{x^2 - 2xy - 8y^2}{16y^2 - x^2}$

$$= \frac{(x-4y)(x+2y)}{-(x^2-16y^2)} = \frac{\cancel{(x-4y)}(x+2y)}{-(\cancel{(x-4y)}(x+4y))}$$

$$= -\frac{(x+2y)}{x+4y}$$

or  $= -\frac{x-2y}{x+4y}$  or  $= -\frac{x+2y}{x+4y}$   $x \neq 4y$   
 $x \neq -4y$

f)  $\frac{x^3 - x^2 + 4x - 4}{x^4 + 3x^2 - 4}$

$$= \frac{x^2(x-1) + 4(x-1)}{(x^2+4)(x^2-1)} = \frac{\cancel{(x-1)}(x^2+4)}{\cancel{(x-1)}(x^2+4)(x+1)}$$

$$= \frac{1}{x+1}, \quad x \neq -1, 1$$

(Note:  $x^2+4 \neq 0$   
 $x^2 = -4$  has no solution  
 $\therefore$  no restriction for this factor)

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**1.6 Multiplying and Dividing Rational Expressions****Rules:** i) express as a product

ii) factor the numerators and denominators fully

iii) state restrictions on the variables

iv) reduce

Ex. Simplify and state restrictions on the variables.

$$\begin{aligned} \text{a) } \frac{12x^3}{7y^4} \times \frac{14y^2}{8x^2} \div \frac{2x^2}{1} \\ = \frac{\overset{3}{\cancel{12}}x^3}{\underset{1}{\cancel{7}}y^4} \cdot \frac{\overset{7}{\cancel{14}}y^2}{\underset{2}{\cancel{8}}x^2} \cdot \frac{1}{\underset{1}{\cancel{2}}x^2} \\ = \frac{3x^3y^2}{2y^4x^4} \\ = \frac{3}{2xy^2}, x \neq 0, y \neq 0 \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{63a^3c^2}{45ab^2} \div \frac{36a^2bc^4}{-15ab^3c} \\ = -\frac{\overset{7}{\cancel{63}}a^3c^2}{\underset{3}{\cancel{45}}ab^2} \cdot \frac{\overset{1}{\cancel{15}}ab^3c}{\underset{4}{\cancel{36}}a^2bc^4} \\ = -\frac{7a^4b^3c^3}{12a^3b^3c^4} \\ = -\frac{7a}{12c}, \begin{matrix} a \neq 0 \\ b \neq 0 \\ c \neq 0 \end{matrix} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{6x-3x^2}{4x+20} \div \frac{9x^2-18x}{3x+15} \\ = \frac{-3x^2+6x}{4x+20} \cdot \frac{3x+15}{9x^2-18x} \\ = \frac{-\overset{1}{\cancel{3}}x(\overset{1}{\cancel{x-2}})}{\underset{1}{\cancel{4}}(\overset{1}{\cancel{x+5}})} \cdot \frac{\overset{3}{\cancel{3}}(\overset{1}{\cancel{x+5}})}{\underset{3}{\cancel{9}}x(\overset{1}{\cancel{x-2}})} \\ = -\frac{1}{4}, x \neq -5, 0, 2 \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{2x+6y}{x^2+7xy+10y^2} \times \frac{x^2+3xy-10y^2}{x^2-4y^2} \\ = \frac{2(x+3y)}{\underset{1}{\cancel{(x+5y)}}(\overset{1}{\cancel{x+2y}})} \cdot \frac{\overset{1}{\cancel{(x+5y)}}(\overset{1}{\cancel{x-2y}})}{\underset{1}{\cancel{(x-2y)}}(\overset{1}{\cancel{x+2y}})} \\ = \frac{2(x+3y)}{(x+2y)^2}, x \neq -5y, -2y, 2y \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{2a^2+a-15}{6a^2+3a} \div \frac{a^3-9a}{2a^2-5a-3} \div \frac{6a^2-11a-10}{3a^2+5a+2} \\ = \frac{2a^2+a-15}{3a(2a+1)} \cdot \frac{2a^2-5a-3}{a(a^2-9)} \cdot \frac{3a^2+5a+2}{6a^2-11a-10} \\ = \frac{\overset{1}{\cancel{(2a-5)}}(\overset{1}{\cancel{(a+3}})})}{\underset{1}{\cancel{3a}}(\overset{1}{\cancel{(2a+1}})})} \cdot \frac{\overset{1}{\cancel{(2a+1)}}(\overset{1}{\cancel{(a-3}})})}{\underset{1}{\cancel{a}}(\overset{1}{\cancel{(a-3}})})(\overset{1}{\cancel{(a+3}})})} \cdot \frac{\overset{1}{\cancel{(3a+2)}}(\overset{1}{\cancel{(a+1}})})}{\underset{1}{\cancel{(3a+2)}}(\overset{1}{\cancel{(2a-5}})})} \\ = \frac{a+1}{3a^2}, a \neq -3, -1, -\frac{2}{3}, -\frac{1}{2}, 0, \frac{5}{2}, 3 \end{aligned}$$

Date: \_\_\_\_\_ 1.7 Adding and Subtracting Rational Expressions

- Rules:** i) fully factor the numerators and denominators of each rational expression stating restrictions on the variables and reducing if possible  
 ii) find the lowest common denominator (LCD) and rewrite each rational expression with the LCD as the denominator for each, and then add or subtract numerators  
 iii) simplify the resulting rational expression, if possible

**Ex. 1.** Simplify and state any restrictions on the variables.

$$\begin{aligned} \text{a) } & \frac{2x-3}{2x} + \frac{1-4x^3}{4x^3} - \frac{5-9x}{6x^2} \quad \text{LCD} = 12x^3 \\ & = \frac{6x^2(2x-3) + 3(1-4x^3) - 2x(5-9x)}{12x^3} \\ & = \frac{12x^3 - 18x^2 + 3 - 12x^3 - 10x + 18x^2}{12x^3} \\ & = \frac{3-10x}{12x^2}, \quad x \neq 0 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{ab^2+2}{2ab^2} - \frac{b+2}{2b} + \frac{2}{1} \quad \text{LCD} = 2ab^2 \\ & = \frac{1(ab^2+2) - ab(b+2) + 2ab^2(2)}{2ab^2} \\ & = \frac{ab^2+2 - ab^2 - 2ab + 4ab^2}{2ab^2} \\ & = \frac{4ab^2 - 2ab + 2}{2ab^2} \\ & = \frac{2(2ab^2 - ab + 1)}{2ab^2} \\ & = \frac{2ab^2 - ab + 1}{ab^2}, \quad a \neq 0, b \neq 0 \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{x+4}{x-3} + \frac{4x-5}{3-x} \\ & = \frac{x+4}{x-3} + \frac{4x-5}{-(x-3)} \\ & = \frac{x+4}{x-3} - \frac{4x-5}{x-3} \quad \text{LCD} = x-3 \\ & = \frac{x+4-4x+5}{x-3} \\ & = \frac{-3x+9}{x-3} \\ & = \frac{-3(\cancel{x-3})}{\cancel{x-3}} \\ & = -3, \quad x \neq 3 \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{2-5x}{x^2-4x+4} - \frac{6x^2-13x-5}{2x^2-9x+10} \\ & = \frac{2-5x}{(x-2)^2} - \frac{(3x+1)(\cancel{2x-5})}{(x-2)(\cancel{2x-5})} \\ & = \frac{2-5x}{(x-2)^2} - \frac{3x+1}{x-2} \quad \text{LCD} = (x-2)(x-2) \\ & = \frac{2-5x - (3x+1)(x-2)}{(x-2)^2} \\ & = \frac{2-5x - (3x^2-5x-2)}{(x-2)^2} \\ & = \frac{2-5x-3x^2+5x+2}{(x-2)^2} \\ & = \frac{-3x^2}{(x-2)^2}, \quad x \neq 2, \frac{5}{2} \end{aligned}$$

Ex. 2. Simplify and state any restrictions on the variables.

$$\begin{aligned}
 \text{a) } & \frac{x+1}{x^2+2x-3} - \frac{x+2}{x^2+4x-5} \\
 &= \frac{x+1}{(x+3)(x-1)} - \frac{x+2}{(x+5)(x-1)} \quad \text{LCD} = (x+5)(x+3)(x-1) \\
 &= \frac{(x+1)(x+5) - (x+2)(x+3)}{(x+5)(x+3)(x-1)} \\
 &= \frac{(x^2+6x+5) - (x^2+5x+6)}{(x+5)(x+3)(x-1)} \\
 &= \frac{x^2+6x+5 - x^2-5x-6}{(x+5)(x+3)(x-1)} \\
 &= \frac{\cancel{x}-1}{(x+5)(x+3)\cancel{(x-1)}} \\
 &= \frac{1}{(x+5)(x+3)}, \quad x \neq -5, -3, 1
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{\left[ \frac{1}{4+m} - \frac{1}{4} \right]}{m} \\
 &= \frac{\left[ \frac{4(1) - 1(4+m)}{4(4+m)} \right]}{\left( \frac{m}{1} \right)} \\
 &= \frac{4-4-m}{4(4+m)} \div \frac{m}{1} \\
 &= \frac{-m}{4(4+m)} \cdot \frac{1}{m} \\
 &= -\frac{1}{4(4+m)}, \quad m \neq -4, 0
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{21}{x^2+7x+10} - \frac{x^2+x-12}{x^2-2x-24} \cdot \frac{x^2+2x-15}{x^2-8x+12} \\
 &= \frac{21}{(x+5)(x+2)} - \frac{\cancel{(x+4)}\cancel{(x-3)}}{\cancel{(x-6)}\cancel{(x+4)}} \cdot \frac{\cancel{(x-6)}(x-2)}{(x+5)\cancel{(x-3)}} \\
 &= \frac{21}{(x+5)(x+2)} - \frac{x-2}{x+5} \quad \text{LCD} = (x+5)(x+2) \\
 &= \frac{21 - (x-2)(x+2)}{(x+5)(x+2)} \\
 &= \frac{21 - (x^2-4)}{(x+5)(x+2)} \\
 &= \frac{21 - x^2 + 4}{(x+5)(x+2)} \\
 &= \frac{25 - x^2}{(x+5)(x+2)} \\
 &= \frac{(5-x)(5+x)}{(x+5)(x+2)} \\
 &= \frac{5-x}{x+2}, \quad x \neq -5, -4, -3, 2, 3, 6
 \end{aligned}$$

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**1.8 Working With Radicals, I**

**Rules:** When **multiplying** radicals,  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ . When **dividing** radicals,  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ .

**Recall:**  $\sqrt{0} = 0$     $\sqrt{4} = 2$     $\sqrt{9} = 3$     $\sqrt{16} = 4$     $\sqrt{25} = 5$     $\sqrt{36} = 6$     $\sqrt{49} = 7$     $\sqrt{64} = 8$     $\sqrt{81} = 9$     $\sqrt{100} = 10$     $\sqrt{121} = 11$     $\sqrt{144} = 12$

**A. Changing Entire Radicals to Mixed Radicals**

1. Simplify.

$$\begin{aligned} \text{a) } \sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{20} &= \sqrt{4 \times 5} \\ &= \sqrt{4} \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt{54} &= \sqrt{9 \times 6} \\ &= \sqrt{9} \sqrt{6} \\ &= 3\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt{48} &= \sqrt{16 \times 3} \\ &= \sqrt{16} \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

**B. Dividing Radicals**

2. Simplify.

$$\begin{aligned} \text{a) } \frac{\sqrt{48}}{\sqrt{6}} &= \sqrt{\frac{48}{6}} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{\sqrt{225}}{\sqrt{3}} &= \sqrt{\frac{225}{3}} \\ &= \sqrt{75} \\ &= 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{2\sqrt{5}}{\sqrt{180}} &= \frac{2\sqrt{5}}{\sqrt{36 \times 5}} \\ &= \frac{2\sqrt{5}}{6\sqrt{5}} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{-10\sqrt{216}}{2\sqrt{3}} &= -5\sqrt{72} \\ &= -5\sqrt{36 \times 2} \\ &= -5(6)\sqrt{2} \\ &= -30\sqrt{2} \end{aligned}$$

**C. Multiplying Radicals**

3. Simplify.

$$\begin{aligned} \text{a) } 2\sqrt{3} \times 5\sqrt{6} &= 2 \times 5 \times \sqrt{3} \times \sqrt{6} \\ &= 10\sqrt{18} \\ &= 10\sqrt{9 \times 2} \\ &= 30\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b) } 9\sqrt{2} \times 4\sqrt{14} &= 36\sqrt{28} \\ &= 36(2)\sqrt{7} \\ &= 72\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{c) } 3\sqrt{7} \times 5\sqrt{7} &= 15\sqrt{49} \\ &= 15(7) \\ &= 105 \end{aligned}$$

**D. Simplifying Radical Expressions**

4. Simplify.

$$\begin{aligned} \text{a) } \frac{21 - 14\sqrt{6}}{7} &= \frac{7(3 - 2\sqrt{6})}{7} \\ &= 3 - 2\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{6 - \sqrt{45}}{-3} &= \frac{6 - 3\sqrt{5}}{-3} \\ &= \frac{-3(2 - \sqrt{5})}{-3} \\ &= -2 + \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{-15 + 2\sqrt{125}}{10} &= \frac{-15 + 2(5\sqrt{5})}{10} \\ &= \frac{-15 + 10\sqrt{5}}{10} \\ &= \frac{5(-3 + 2\sqrt{5})}{10} \\ &= \frac{-3 + 2\sqrt{5}}{2} \end{aligned}$$

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**1.9 Working With Radicals, II****A. Adding and Subtracting Radicals**1. Simplify (ie. add and subtract *like* radicals).

$$\begin{aligned} \text{a) } & \sqrt{24} - \sqrt{54} + \sqrt{150} \\ & = 2\sqrt{6} - 3\sqrt{6} + 5\sqrt{6} \\ & = 4\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b) } & 3\sqrt{48} - \frac{1}{2}\sqrt{8} + 2\sqrt{27} - \frac{4}{3}\sqrt{72} \\ & = 3(4\sqrt{3}) - \frac{1}{2}(2\sqrt{2}) + 2(3\sqrt{3}) - \frac{4}{3}(6\sqrt{2}) \\ & = 12\sqrt{3} - \sqrt{2} + 6\sqrt{3} - 8\sqrt{2} \\ & = 18\sqrt{3} - 9\sqrt{2} \end{aligned}$$

**B. Multiplying Radicals**

2. Simplify (ie. expand first and then simplify.)

$$\begin{aligned} \text{a) } & 2\sqrt{2}(\sqrt{10} - 3\sqrt{14}) \\ & = 2\sqrt{20} - 6\sqrt{28} \\ & = 2(2\sqrt{5}) - 6(2\sqrt{7}) \\ & = 4\sqrt{5} - 12\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{b) } & (2\sqrt{3} + \sqrt{5})(\sqrt{3} - 4\sqrt{5}) \\ & = 2\sqrt{9} - 8\sqrt{15} + \sqrt{15} - 4\sqrt{25} \\ & = 2(3) - 7\sqrt{15} - 4(5) \\ & = 6 - 20 - 7\sqrt{15} \\ & = -14 - 7\sqrt{15} \end{aligned}$$

**C. Dividing Radicals**3. Simplify (ie. *rationalize* the denominator).

$$\begin{aligned} \text{a) } & \frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \\ & = \frac{5\sqrt{6}}{\sqrt{36}} \\ & = \frac{5\sqrt{6}}{6} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{\sqrt{5}}{\sqrt{40}} \\ & = \frac{1}{\sqrt{8}} \\ & = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ & = \frac{\sqrt{2}}{2\sqrt{4}} \\ & = \frac{\sqrt{2}}{4} \end{aligned}$$

Recall:  $(a-b)(a+b) = a^2 - b^2$ 

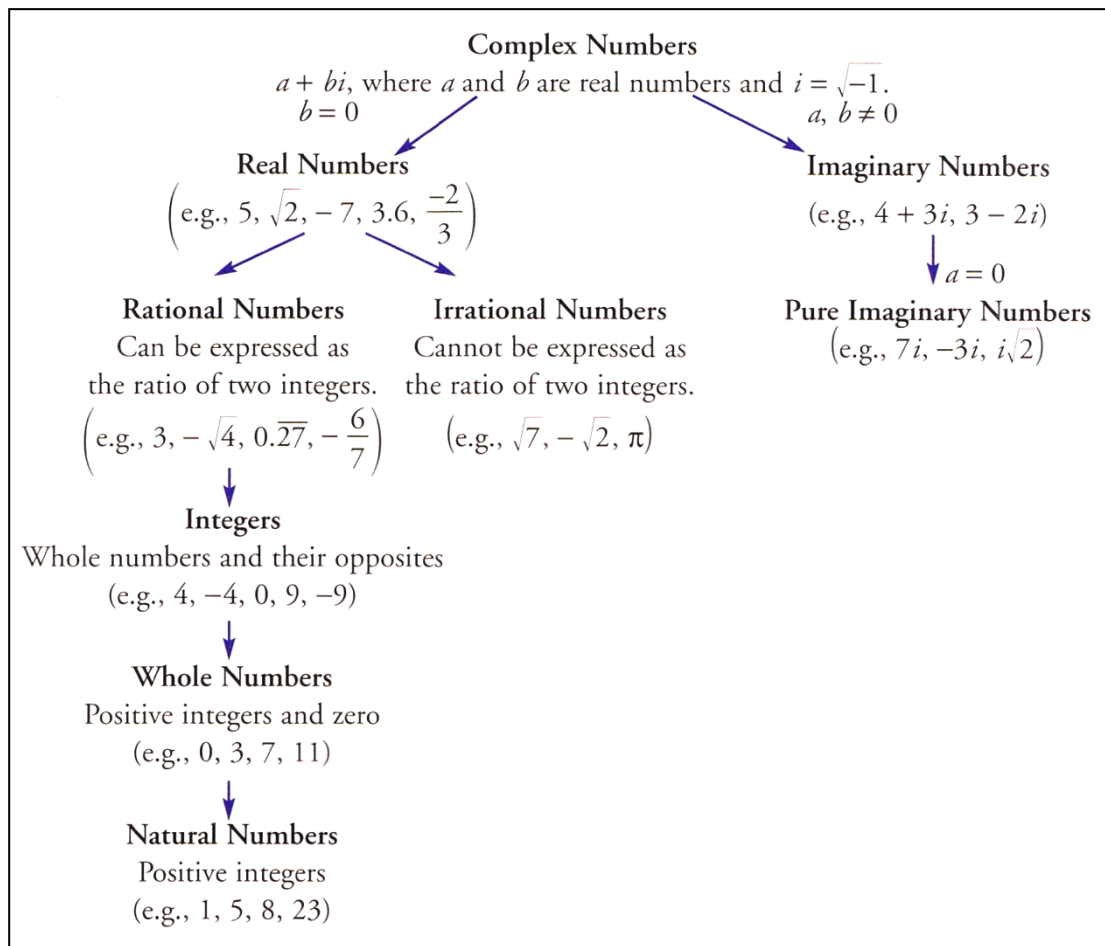
$$\begin{aligned} \text{c) } & \frac{3}{2\sqrt{6} - \sqrt{3}} \cdot \frac{2\sqrt{6} + \sqrt{3}}{2\sqrt{6} + \sqrt{3}} \\ & = \frac{6\sqrt{6} + 3\sqrt{3}}{4\sqrt{36} - 9} \\ & = \frac{6\sqrt{6} + 3\sqrt{3}}{4(6) - 9} \\ & = \frac{6\sqrt{6} + 3\sqrt{3}}{21} \\ & = \frac{3(2\sqrt{6} + \sqrt{3})}{21} \\ & = \frac{2\sqrt{6} + \sqrt{3}}{7} \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{2\sqrt{7} - \sqrt{3}}{2\sqrt{3} + 3\sqrt{7}} \cdot \frac{2\sqrt{3} - 3\sqrt{7}}{2\sqrt{3} - 3\sqrt{7}} \\ & = \frac{4\sqrt{21} - 6\sqrt{49} - 2\sqrt{9} + 3\sqrt{21}}{4\sqrt{9} - 9\sqrt{49}} \\ & = \frac{7\sqrt{21} - 6(7) - 2(3)}{4(3) - 9(7)} \\ & = \frac{7\sqrt{21} - 48}{-51} \\ & = \frac{-7\sqrt{21} + 48}{51} \end{aligned}$$

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## 1.10 Working With Complex Numbers

**Definition:** Complex Numbers,  $C$ , are of the form  $a + bi$ , where  $a$  and  $b$  are real numbers and  $i$ , the imaginary unit is equal to  $\sqrt{-1}$ , ie.  $i = \sqrt{-1}$  and  $i^2 = -1$ .



### A. Simplifying Complex Numbers

#### 1. Simplify.

a)  $\sqrt{-25}$   
 $= \sqrt{25} \sqrt{-1}$   
 $= 5i$

b)  $\sqrt{-48}$   
 $= \sqrt{16} \sqrt{-1} \sqrt{3}$   
 $= 4i\sqrt{3}$

c)  $\sqrt{-125}$   
 $= \sqrt{25} \sqrt{-1} \sqrt{5}$   
 $= 5i\sqrt{5}$

d)  $-\sqrt{-4}$   
 $= -\sqrt{4} \sqrt{-1}$   
 $= -2i$

e)  $3 - \sqrt{-28}$   
 $= 3 - 2i\sqrt{7}$

f)  $\frac{-21 + \sqrt{-98}}{7}$   
 $= \frac{-21 + 7i\sqrt{2}}{7}$   
 $= \frac{\cancel{7}(-3 + i\sqrt{2})}{\cancel{7}}$   
 $= -3 + i\sqrt{2}$

g)  $\frac{-12 + \sqrt{-9}}{-6}$   
 $= \frac{-12 + 3i}{-6}$   
 $= \frac{\cancel{-3}(4 - i)}{\cancel{-6}}$   
 $= \frac{4 - i}{2}$

## B. Adding and Subtracting Complex Numbers

2. Simplify.

$$\begin{aligned} \text{a) } (6-4i)+(-2+3i) \\ &= 6-2-4i+3i \\ &= 4-i \end{aligned}$$

$$\begin{aligned} \text{b) } (-4-5i)-(3-2i) \\ &= -4-3-5i+2i \\ &= -7-3i \end{aligned}$$

## C. Multiplying Complex Numbers *Recall: If $i=\sqrt{-1}$ then $i^2=-1$*

3. Simplify.

$$\begin{aligned} \text{a) } 3i \times 6i \\ &= 18i^2 \\ &= 18(-1) \\ &= -18 \end{aligned}$$

$$\begin{aligned} \text{b) } (2i)(-5i)(3i) \\ &= -30i^3 \\ &= -30i^2 \cdot i \\ &= -30(-1)i \\ &= 30i \end{aligned}$$

$$\begin{aligned} \text{c) } (-3i\sqrt{2})^2 \\ &= 9i^2(2) \\ &= 18i^2 \\ &= 18(-1) \\ &= -18 \end{aligned}$$

$$\begin{aligned} \text{d) } 3i(2+4i) \\ &= 6i+12i^2 \\ &= 6i+12(-1) \\ &= -12+6i \end{aligned}$$

$$\begin{aligned} \text{e) } (1-4i)(3+2i) \\ &= 3+2i-12i-8i^2 \\ &= 3-10i-8(-1) \\ &= 3+8-10i \\ &= 11-10i \end{aligned}$$

$$\begin{aligned} \text{f) } (3-5i)^2 \\ &= (3-5i)(3-5i) \\ &= 9-15i-15i+25i^2 \\ &= 9-30i+25(-1) \\ &= 9-25-30i \\ &= -16-30i \end{aligned}$$

## D. Dividing Complex Numbers

4. Simplify by *rationalizing* the denominators.

$$\begin{aligned} \text{a) } \frac{5}{2i} \cdot \frac{i}{i} \\ &= \frac{5i}{2i^2} \\ &= \frac{5i}{2(-1)} \\ &= -\frac{5i}{2} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{3i}{-3+i} \\ &= \frac{3i}{i-3} \cdot \frac{i+3}{i+3} \\ &= \frac{3i^2+9i}{i^2-3^2} \\ &= \frac{3(-1)+9i}{(-1)-9} \\ &= \frac{-3+9i}{-10} \\ &= \frac{3-9i}{10} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{2+3i}{1-2i} \cdot \frac{1+2i}{1+2i} \\ &= \frac{2+2i+3i+6i^2}{1-4i^2} \\ &= \frac{2+5i+6(-1)}{1-4(-1)} \\ &= \frac{2-6+5i}{1+4} \\ &= \frac{-4+5i}{5} \end{aligned}$$

HW. Exercise 1.10

Unit 1 Part II Test covers Days 5 to 10

HW. Part II Review 1.5 to 1.10