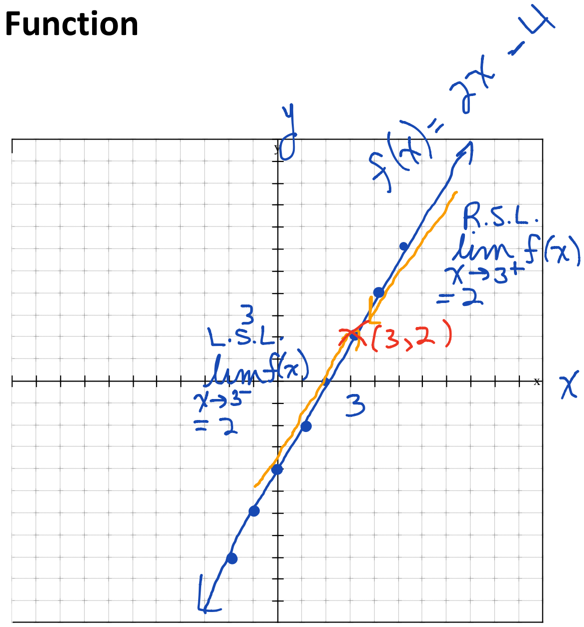
Date: Feb. 10/14

Section 3.3 – The Limit of a Function

Warm-up

1. Graph $f(x) = 2x - 4$.



2. Use the ***graph*** to evaluate the following limit.

$\lim_{x \to 3} f(x)$

$\because \lim_{x\to 3^-} f(x) = 2 = \lim_{x\to 3^+} f(x)$

$\therefore \lim_{x\to 3} f(x) = 2$

3. Evaluate the same limit ***algebraically***.

$\lim_{x\to 3} f(x)$

$= \lim_{x\to 3} (2x-4)$

$= 2(3) - 4$

$= 2$

Ex. 1. Evaluate the following limits by using the ***direct substitution technique***.

a) $\lim_{x\to -1}(-x^2 + 2x - 4)$

$= -(-1)^2 + 2(-1) - 4$

$= -7$

b) $\lim_{x\to 2} \frac{-4}{x+4}$

$= \frac{-4}{(2)+4}$

$= -\frac{2}{3}$

c) $\lim_{x\to 3^+} \sqrt{x^2 - 9}$

$= \sqrt{(3)^2 - 9}$

$= 0$

Evaluating the Limit of a Function Using One-Sided Limits

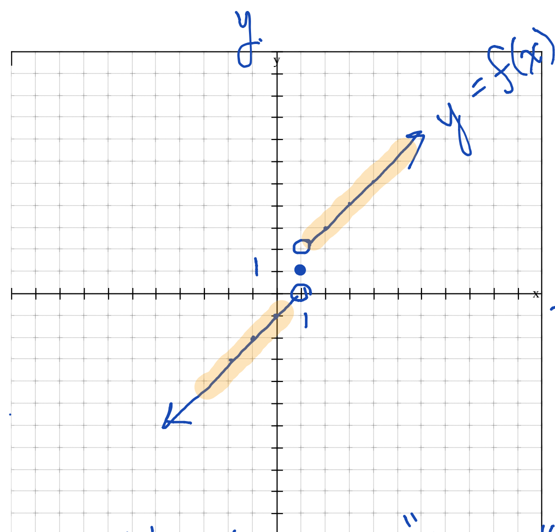
1. Graph $f(x) = \begin{cases} x-1, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ x+1, & \text{if } x > 1 \end{cases}$

If $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$, then $\lim_{x\to a} f(x)$ does not exist.

If $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$, then $\lim_{x\to a} f(x) = L$.

2. Use the ***graph*** to evaluate ***one-sided limits***, in order to find the indicated limit if it exists.

$\lim_{x\to 1} f(x)$



Left-sided limit

$\lim_{x\to 1^-} f(x)$

$= 0$

Right-sided limit

$\lim_{x\to 1^+} f(x)$

$= 2$

$\because \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) \therefore \lim_{x\to 1} f(x)$ does not exist

3. Evaluate the same limit ***algebraically***, using ***one-sided limits***.

$\lim_{x\to 1^-} f(x)$

$= \lim_{x\to 1^-}(x-1)$

$= 1 - 1$

$= 0$

$\lim_{x\to 1^+} f(x)$

$= \lim_{x\to 1^+}(x+1)$

$= 1+1$

$= 2$

Note: There is a "jump" discontinuity at $x = 1$.

$\because \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$, $\therefore \lim_{x\to 1} f(x)$ does not exist.

KEY CONCEPTS

- The limit of a function is written as $\lim_{x \to a} f(x) = L$, which is read as "the limit of $f(x)$ as x approaches a, equals L".

- If the values of $f(x)$ approach L more and more closely as x approaches a more and more closely (from either side of a), but $x \neq a$, then $\lim_{x \to a} f(x) = L$.

- The left-sided limit of a function is written as $\lim_{x \to a^-} f(x)$, which is read as "the limit of $f(x)$ as x approaches a from the left".

- If the values of $f(x)$ approach L more and more closely as x approaches a more and more closely, with $x < a$, then $\lim_{x \to a^-} f(x) = L$.

- The right-sided limit of a function is written as $\lim_{x \to a^+} f(x)$, which is read as "the limit of $f(x)$ as x approaches a from the right".

- If the values of $f(x)$ approach L more and more closely as x approaches a more and more closely, with $x > a$, then $\lim_{x \to a^+} f(x) = L$.

- In order for $\lim_{x \to a} f(x)$ to exist, the one-sided limits $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ must both exist and be equal. That is,

  If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.

  If $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x) = L$.

- To check that a function $f(x)$ is continuous at $x = a$, check that the following three conditions are satisfied:

  a) $f(a)$ is defined (a is in the domain of $f(x)$)
  b) $\lim_{x \to a} f(x)$ exists
  c) $\lim_{x \to a} f(x) = f(a)$

  Also, if $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

- Every polynomial P is continuous at every number, that is, $\lim_{x \to a} P(x) = P(a)$.

- Every rational function $f(x) = \dfrac{P(x)}{Q(x)}$ where P and Q are polynomials, is continuous at every number a for which $Q(a) \neq 0$, that is, $\lim_{x \to a} \dfrac{P(x)}{Q(x)} = \dfrac{P(a)}{Q(a)}, Q(a) \neq 0$.

- Discontinuous:
  a) If a function $f(x)$ has a removable discontinuity at $x = a$, then $\lim_{x \to a} f(x) = L$ exists, and the discontinuity can be removed by (re)defining $f(x) = L$ at the single point a.
  b) If a function has a jump discontinuity, the function "jumps" from one value to another.
  c) If a function $f(x)$ has in infinite discontinuity at $x = a$, the absolute values of the function become larger and larger as x approaches a.

Ex. 2. By evaluating ***one-sided limits***, find the indicated limit if it exists. Graph the function and state whether the function is continuous or discontinuous with reasons.

Note: A function that is ***continuous*** has no breaks in its graph.
A function that is ***discontinuous*** has some type of break in its graph. This break is the result of a ***hole***, ***jump***, or ***vertical asymptote***.

a) $f(x)=\begin{cases}2x+4, & \text{if } x<-2\\ 4-x^2, & \text{if } x\geq -2\end{cases}$; $\lim\limits_{x\to -2} f(x)$

L.S.L.

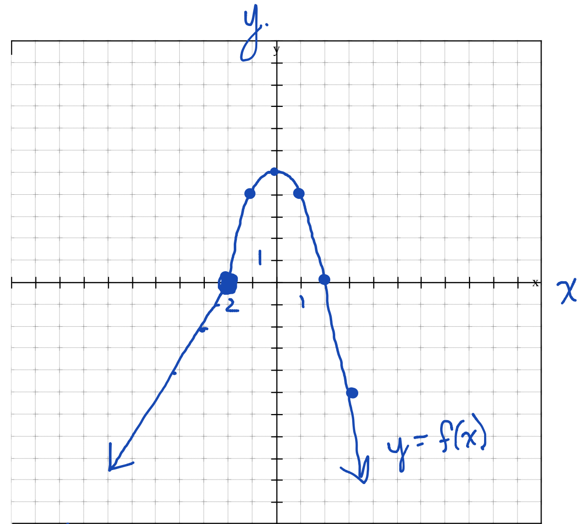
$\lim\limits_{x\to -2^-} f(x)$
$= \lim\limits_{x\to -2^-} (2x+4)$
$= 2(-2)+4$
$= 0$

R.S.L.

$\lim\limits_{x\to -2^+} f(x)$
$= \lim\limits_{x\to -2^+} (4-x^2)$
$= 4-(-2)^2$
$= 0$

$\because \lim\limits_{x\to -2^-} f(x) = 0 = \lim\limits_{x\to 2^+} f(x)$

$\therefore \lim\limits_{x\to -2} f(x) = 0$



Note: $f(x)$ is continuous for all $x \in \mathbb{R}$.

b) $g(x)=\begin{cases}-x+3, & \text{if } x\geq 1\\ x^3, & \text{if } x<1\end{cases}$; $\lim\limits_{x\to 1} g(x)$

L.S.L.

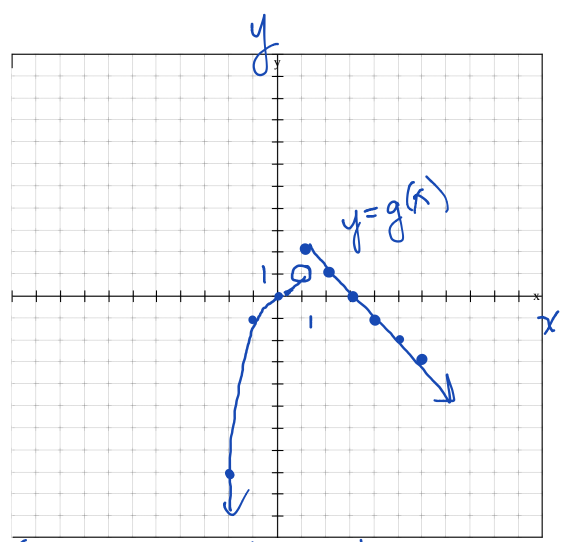
$\lim\limits_{x\to 1^-} g(x)$
$= \lim\limits_{x\to 1^-} x^3$
$= (1)^3$
$= 1$

R.S.L.

$\lim\limits_{x\to 1^+} g(x)$
$= \lim\limits_{x\to 1^+} (-x+3)$
$= -(1)+3$
$= 2$

$\because \lim\limits_{x\to 1^-} g(x) \neq \lim\limits_{x\to 1^+} g(x)$

$\therefore \lim\limits_{x\to 1} g(x)$ does not exist.

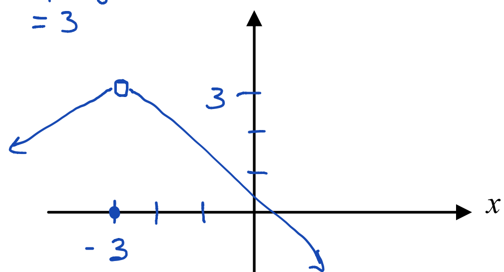


Note: $g(x)$ is discontinuous at $x=1$ where there is a "jump" discontinuity.

Ex. 3. Sketch the graph of any function that satisfies the given conditions in each case.

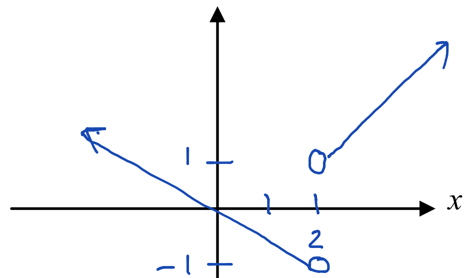
a) $\lim_{x\to -3} f(x) = 3,\ f(-3) = 0$ ← pt. $(-3,0)$

$\lim_{x\to -3^-} f(x) = 3$ & $\lim_{x\to -3^+} f(x) = 3$



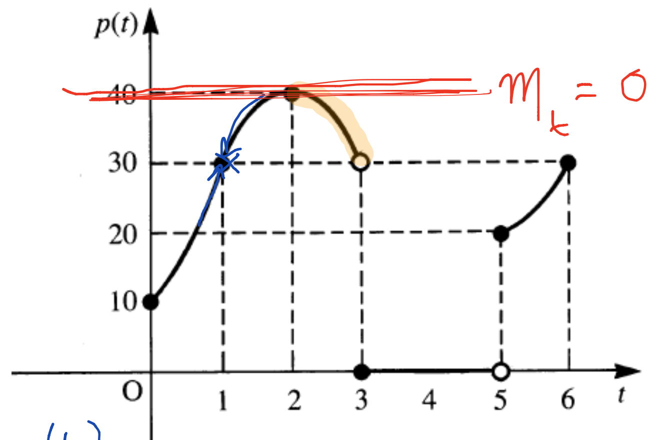
Note: There is a removable discontinuity at $x=-3$.

b) $\lim_{x\to 2^-} f(x) = -1,\ \lim_{x\to 2^+} f(x) = 1$



Note: There is a "jump" discontinuity at $x=2$.

Ex. 4. The function $p(t)$ describes the production of unleaded gasoline in a refinery, in thousands of litres, where the time t, is measured in days.



a) Evaluate $\lim_{t\to 1} p(t)$.

$\lim_{t\to 1} p(t) = 30$

b) Evaluate $\lim_{t\to 3^-} p(t)$ and $\lim_{t\to 3^+} p(t)$.

$\lim_{t\to 3^-} p(t) = 30$ $\qquad$ $\lim_{t\to 3^+} p(t) = 0$

c) When was the refinery shut down for repairs and when did production begin again?

The refinery shut down on day 3 and opened on day 5.

d) At what times is the production function $p(t)$ discontinuous?

$p(t)$ is discontinuous at $t=3$ and $t=5$.

e) At what time was the production highest and what was the rate of change of production at this time?

Production was highest at $t=2$ where the rate of change of production was 0.

HW: p. 98 #4 to 8, 10 to 14

Date: Feb. 11 / 14

Warm-up

1. Evaluate each limit, if possible, using the ***direct substitution technique*** for evaluating limits.

a) $\lim_{x\to 4}\left(\sqrt{x}+2\right)^2$

$= \left(\sqrt{4}+2\right)^2$
$= (2+2)^2$
$= (4)^2$
$= 16$

b) $\lim_{x\to 0}\left(\sqrt{1+\sqrt{1+x}}\right)$

$= \sqrt{1+\sqrt{1}}$
$= \sqrt{1+1}$
$= \sqrt{2}$

c) $\lim_{x\to a}\frac{(x+a)^2}{x^2+a^2}$

$= \frac{(a+a)^2}{a^2+a^2}$
$= \frac{(2a)^2}{2a^2}$
$= \frac{4a^2}{2a^2}$ $= 2$

d) $\lim_{x\to -2}\frac{1}{x+2}$

$= \frac{1}{-2+2}$
$= \frac{1}{0}$
undefined or d.n.e.

e) $\lim_{x\to 4} f(x)$ if $f(x)=\begin{cases} x+3, & \text{if } x>2 \\ \frac{4}{x}, & \text{if } x\le 2\end{cases}$

$= \lim_{x\to 4}(x+3)$
$= (4)+3$
$= 7$

f) $\lim_{x\to -\frac{1}{2}} f(x)$ if $f(x)=\begin{cases} x+3, & \text{if } x>2 \\ \frac{4}{x}, & \text{if } x\le 2\end{cases}$

$= \lim_{x\to -\frac{1}{2}} \frac{4}{x}$
$= 4 \div \left(-\frac{1}{2}\right)$
$= -\frac{4}{1}\times\frac{2}{1}$ $= -8$

2. By evaluating ***one-sided limits***, find the indicated limit if it exists.
Graph the function and state whether the function is continuous.

$f(x)=\begin{cases} x+3, & \text{if } x>2 \\ \frac{4}{x}, & \text{if } x\le 2\end{cases}$; $\lim_{x\to 2} f(x)$

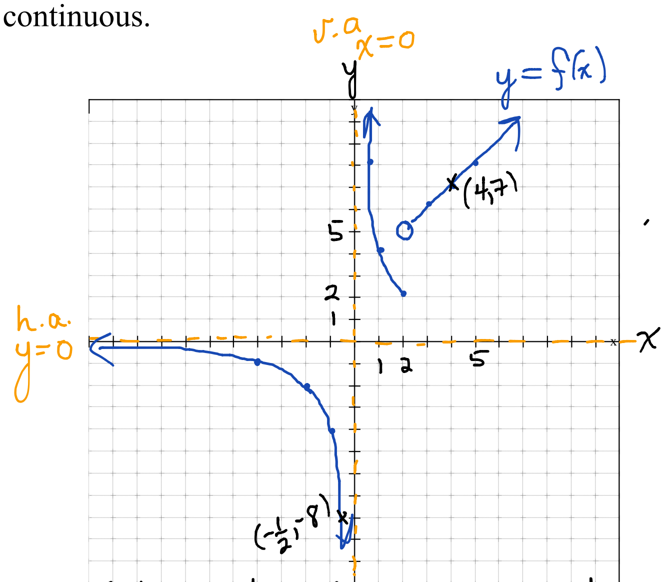
L.S.L.

$\lim_{x\to 2^-} f(x)$
$= \lim_{x\to 2^-}\frac{4}{x}$
$= \frac{4}{2}$
$= 2$

R.S.L.

$\lim_{x\to 2^+} f(x)$
$= \lim_{x\to 2^+}(x+3)$
$= (2)+3$
$= 5$

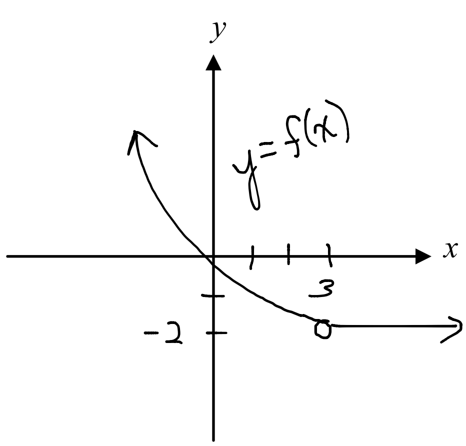
$\because \lim_{x\to 2^-} f(x) \ne \lim_{x\to 2^+} f(x)$
$\therefore \lim_{x\to 2} f(x)$ does not exist



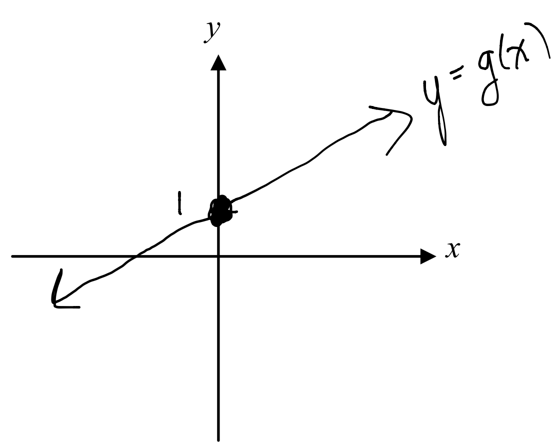
Note: The function is discontinuous at $x=0$ where there is an infinite discontinuity & at $x=2$ where there is a "jump" discontinuity.

3. Sketch the graph of any function that satisfies the given conditions in each case.

a) $\lim_{x\to 3} f(x)=-2$, $f(x)$ is discontinuous at $x=3$



b) $\lim_{x\to 0} g(x)=1$, $g(x)$ is continuous at $x=0$



Date: Feb. 11/14

Section 3.4 – Limits of Indeterminate Forms

Sometimes the limit of $f(x)$ as x approaches a cannot be found by direct substitution. This is of special interest when direct substitution results in an ***indeterminate form*** $\left(\frac{0}{0}\right)$. In such cases, we look for an equivalent function that agrees with f for all values except the troublesome value at $x = a$.

Recall: Factoring

Difference of squares	***Difference of cubes***	***Sum of cubes***
$a^2 - b^2$	$a^3 - b^3$	$a^3 + b^3$
$= (a-b)(a+b)$	$= (a-b)(a^2+ab+b^2)$	$= (a+b)(a^2-ab+b^2)$

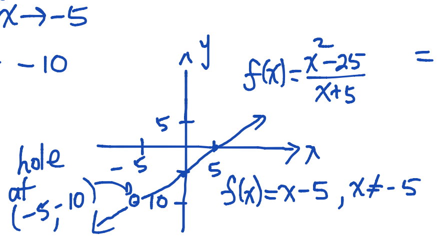
Ex. 1. Evaluate the limit of each ***indeterminate*** quotient by using ***factoring*** or ***rationalizing techniques***.

a) $\lim\limits_{x\to -5} \dfrac{x^2-25}{x+5}$ $\left(\frac{0}{0}\right)$

$= \lim\limits_{x\to -5} \dfrac{(x-5)\cancel{(x+5)}}{\cancel{(x+5)}}$

$= \lim\limits_{x\to -5} (x-5)$

$= -10$



b) $\lim\limits_{x\to -3} \dfrac{x^3+27}{3x+9}$ $\left(\frac{0}{0}\right)$

$= \lim\limits_{x\to -3} \dfrac{x^3+3^3}{3(x+3)}$

$= \lim\limits_{x\to -3} \dfrac{\cancel{(x+3)}(x^2-3x+9)}{3\cancel{(x+3)}}$

$= \lim\limits_{x\to -3} \dfrac{x^2-3x+9}{3}$

$= \dfrac{9+9+9}{3}$

$= 9$

c) $\lim\limits_{x\to 0} \dfrac{3^{2x}-3^x}{1-3^x}$ $\left(\frac{0}{0}\right)$

$= \lim\limits_{x\to 0} \dfrac{3^x(3^x-1)}{-3^x+1}$

$= \lim\limits_{x\to 0} \dfrac{3^x\cancel{(3^x-1)}}{-\cancel{(3^x-1)}}$

$= \lim\limits_{x\to 0} \dfrac{3^x}{-1}$

$= -1$

d) $\lim\limits_{x\to \frac{3}{2}} \dfrac{8x^3-27}{2x^3-9x^2+x+12}$

$$
\begin{array}{r}
x^2-3x-4 \\
2x-3\,\big)\,2x^3-9x^2+x+12 \\
2x^3-3x^2 \\
\hline
-6x^2+x \\
-6x^2+9x \\
\hline
-8x+12 \\
-8x+12 \\
\hline
0
\end{array}
$$

$= \lim\limits_{x\to \frac{3}{2}} \dfrac{(2x)^3-(3)^3}{(2x-3)(x^2-3x-4)}$

$= \lim\limits_{x\to \frac{3}{2}} \dfrac{\cancel{[(2x)-(3)]}[(2x)^2+(2x)(3)+(3)^2]}{\cancel{(2x-3)}(x^2-3x-4)}$

$= \lim\limits_{x\to \frac{3}{2}} \dfrac{4x^2+6x+9}{x^2-3x-4}$

$= \dfrac{\cancel{4}\left(\frac{9}{\cancel{4}}\right)+\cancel{6}^{3}\left(\frac{3}{\cancel{2}}\right)+9}{\frac{9}{4}-\frac{9}{2}-4}$

$= \dfrac{27}{-\frac{25}{4}}$

$\left\{ \frac{9}{4}-\frac{18}{4}-\frac{16}{4} \right.$

$= -27 \times \dfrac{4}{25}$

$= \dfrac{-108}{25}$

e) $\lim\limits_{x\to 0} \dfrac{1-\sqrt{1+3x}}{x}$ $\left(\frac{0}{0}\right)$

$= \lim\limits_{x\to 0} \dfrac{(1-\sqrt{1+3x})}{x} \cdot \dfrac{(1+\sqrt{1+3x})}{(1+\sqrt{1+3x})}$

$= \lim\limits_{x\to 0} \dfrac{1-(1+3x)}{x(1+\sqrt{1+3x})}$

$= \lim\limits_{x\to 0} \dfrac{-3\cancel{x}}{\cancel{x}(1+\sqrt{1+3x})}$

$= \lim\limits_{x\to 0} \dfrac{-3}{1+\sqrt{1+3x}}$

$= -\dfrac{3}{2}$

Ex. 2. Evaluate the following limit using the ***change of variable technique***.

$$\lim_{\boxed{x\to 1}}\frac{\boxed{x^{\frac{1}{6}}}-1}{\boxed{x}-1} \quad \left(\frac{0}{0}\right)$$

Let $u = x^{\frac{1}{6}}$

If $x \to 1$, $u \to (1)^{\frac{1}{6}}$

$u \to \sqrt[6]{1}$

$u \to 1$

Find x if

$(u)^6 = \left(x^{\frac{1}{6}}\right)^6$

$u^6 = x$

$\therefore x = u^6$

$$= \lim_{u\to 1}\frac{u-1}{u^6-1} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{u\to 1}\frac{u-1}{\underbrace{(u^3-1)}(u^3+1)}$$

$$= \lim_{u\to 1}\frac{\cancel{(u-1)}\,1}{\cancel{(u-1)}(u^2+u+1)(u^3+1)}$$

$$= \lim_{u\to 1}\frac{1}{(u^2+u+1)(u^3+1)}$$

$$= \frac{1}{(3)(2)}$$

$$= \frac{1}{6}$$

Ex. 3. Evaluate the following limit using the ***one-sided limits technique***.
Illustrate your results graphically.

$$\lim_{x\to 2}\frac{x|x-2|}{x-2} \quad \left(\frac{0}{0}\right)$$

Case ①

If $x-2<0$

$x<2$

L.S.L.

$$= \lim_{x\to 2^-}\frac{x\left[-\cancel{(x-2)}\right]}{\cancel{x-2}}$$

$$= \lim_{x\to 2^-}(-x)$$

$$= -2$$

Case ②

If $x-2>0$

$x>2$

R.S.L.

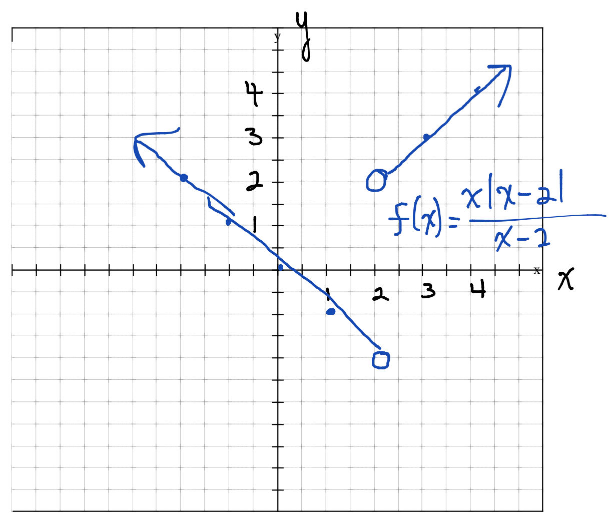
$$= \lim_{x\to 2^+}\frac{x\left[+\cancel{(x-2)}\right]}{\cancel{x-2}}$$

$$= \lim_{x\to 2^+} x$$

$$= 2$$

$\therefore$ L.S.L. $\neq$ R.S.L.

$\therefore \lim_{x\to 2}\frac{x|x-2|}{x-2}$ does not exist.



Let $f(x) = \frac{x|x-2|}{x-2}$

$$f(x) = \begin{cases} -x & , \text{if } x<2 \\ x & , \text{if } x>2 \end{cases}$$

HW: p. 105 #1, 4, 7 to 10

Date: Feb. 12/14

Section 3.4 – Properties of Limits

The Limit Laws

Suppose that the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and c is a constant.

Then, we have the following limit laws.

1. $\lim_{x\to a} c = c$
2. $\lim_{x\to a} x = a$
3. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ -The limit of a sum is the sum of the limits. "Sum Law"
4. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ -The limit of a difference is the difference of the limits. "Difference Law"
5. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$ -The limit of a constant times a function is the constant times the limit of the function. "Constant Multiple Law"
6. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\ \lim_{x\to a} g(x)$ -The limit of a product is the product of the limits. "Product Law"
7. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ *if* $\lim_{x\to a} g(x) \neq 0$ -The limit of a quotient is the quotient of the limits. "Quotient Law"
8. $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ -The limit of a power is the power of the limit. "Power Law"
9. $\lim_{x\to a} x^n = a^n$
10. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ -The limit of a root is the root of the limit. "Root Law"
11. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

Ex. 1. If $\boxed{\lim_{x\to -2} f(x) = 9}$, state and use the ***properties of limits*** to evaluate each limit.

a) $\lim_{x\to -2} 2[f(x)]^2$

$= 2\cdot \lim_{x\to -2}[f(x)]^2$ ← by the Constant Multiple Law

$= 2\cdot\left[\lim_{x\to -2} f(x)\right]^2$ ← by the Power Law

$= 2(9)^2$

$= 162$

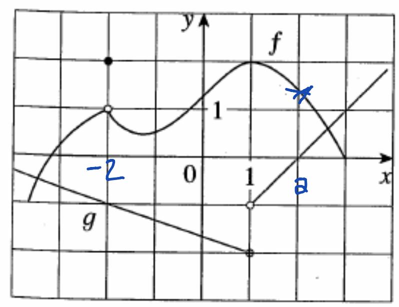
b) $\lim_{x\to -2} \dfrac{\sqrt{f(x)}}{f(x)-x^2}$

$= \dfrac{\lim_{x\to -2}\sqrt{f(x)}}{\lim_{x\to -2}[f(x)-x^2]}$ ← by the Quotient Law

$= \dfrac{\sqrt{\lim_{x\to -2} f(x)}}{\lim_{x\to -2} f(x) - \lim_{x\to -2} x^2}$ ← by the Root Law ← by the Difference Law

$= \dfrac{\sqrt{9}}{9-(4)} = \dfrac{3}{5}$

Ex. 2. Given the graphs of f and g, use and state the ***Limit Laws*** to evaluate the following limits if they exist.



a) $\lim_{x\to 2}\frac{f(x)}{g(x)}$

$= \dfrac{\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$ ← by the quotient law

$\doteq \dfrac{1.5}{0}$ $\because \lim_{x\to 2} g(x) = 0$ $\therefore \lim_{x\to 2}\frac{f(x)}{g(x)}$ d.n.e.

b) $\lim_{x\to -2}[3f(x)+g(x)]$

$= \lim_{x\to -2} 3f(x) + \lim_{x\to -2} g(x)$ ← by the sum law

$= 3\lim_{x\to -2} f(x) + \lim_{x\to -2} g(x)$ ← by the constant multiple Law.

$= 3(1) + (-1)$

$= 2$

c) $\lim_{x\to 1^-}[f(x)\,g(x)]$

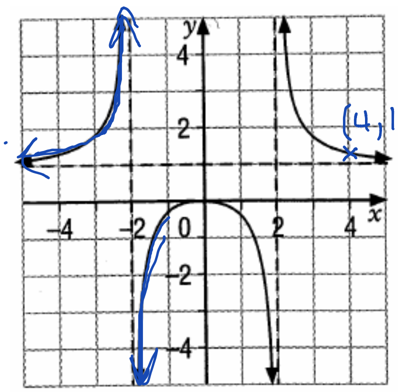
$= \lim_{x\to 1^-} f(x)\cdot\lim_{x\to 1^-} g(x)$ ← by the Product Law

$= (2)(-2)$

$= -4$

Ex. 3. Evaluate the following limits if possible. (Use $+\infty$ or $-\infty$ if appropriate.)

$f(x) = \dfrac{x^2}{x^2-4}$



a) $\lim_{x\to 4}\frac{x^2}{x^2-4}$

$= \dfrac{(4)^2}{(4)^2-4}$

$= \dfrac{16}{12}$

$= \dfrac{4}{3}$

b) $\lim_{x\to -\infty}\frac{x^2}{x^2-4}$

$= 1$

c) $\lim_{x\to +\infty}\frac{x^2}{x^2-4}$ $\div \frac{x^2}{x^2}$

$= \lim_{x\to +\infty}\dfrac{1}{1-\frac{4}{x^2}}$

$= \dfrac{1}{1-0}$

$= 1$

d) $\lim_{x\to -2^-}\frac{x^2}{x^2-4}$

$= +\infty$

if $x=-1.9$ $\dfrac{(+)}{(-)(+)}$

e) $\lim_{x\to -2^+}\frac{x^2}{x^2-4}$ $\frac{4}{0}$

$= \lim_{x\to -2^+}\dfrac{x^2}{(x-2)(x+2)}$

$= \boxed{-}\infty$

f) $\lim_{x\to -2}\frac{x^2}{x^2-4}$

$=$ d.n.e.

$\because \lim_{x\to -2^-}\frac{x^2}{x^2-4} \neq \lim_{x\to -2^+}\frac{x^2}{x^2-4}$

Ex. 4. Evaluate the following limits if possible. (Use $+\infty$ or $-\infty$ if appropriate.)

a) $\lim_{x\to\infty}\frac{6x^2+3x-2}{(3x-2)^2}$

$= \lim_{x\to\infty}\dfrac{6x^2+3x-2}{9x^2-12x+4}$ $\div\frac{x^2}{x^2}$

$= \lim_{x\to\infty}\dfrac{6+\frac{3}{x}-\frac{2}{x^2}}{9-\frac{12}{x}+\frac{4}{x^2}}$

$= \dfrac{6}{9}$

$= \dfrac{2}{3}$

b) $\lim_{x\to -\infty}\frac{x^2+x}{3-x}$ $\div\frac{x}{x}$

$-x+3\,\overline{)\,x^2+x+0}$ quotient $-x-4$; x^2-3x; $4x+0$; $4x-12$; 12

$= \lim_{x\to -\infty}\left(-x-4+\dfrac{12}{3-x}\right)$

$= +\infty$

or

$= \lim_{x\to -\infty}\dfrac{x+1}{\frac{3}{x}-1}$

$= +\infty$

L.O.A. $y=-x-4$

c) $\lim_{x\to\infty}\sqrt{x^2+1}$

$= +\infty$

d) $\lim_{x\to 3}\frac{x^2-9}{x^2+2x-3}$ 12

$= \dfrac{0}{12}$

$= 0$

e) $\lim_{x\to -3}\frac{x^2-9}{x^2+2x-3}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to -3}\dfrac{(x-3)(x+3)}{(x+3)(x-1)}$

$= \lim_{x\to -3}\dfrac{x-3}{x-1}$

$= \dfrac{-6}{-4}$

$= \dfrac{3}{2}$

f) $\lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3}$ $\frac{-8}{0}$

$= \lim_{x\to 1^+}\dfrac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)}$

$= \boxed{-}\infty$

$x=1.1$ $\dfrac{(-)}{(+)}$

HW: p. 107 #13, 16 to 19 & Addition Limits Worksheet #1 to 3 odd parts

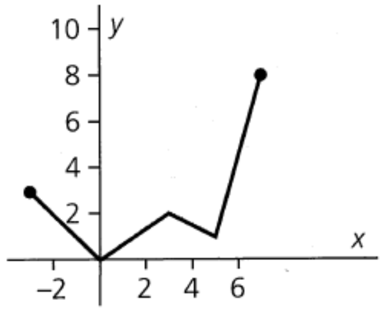
a c e . . . .

Date: Feb. 13/14

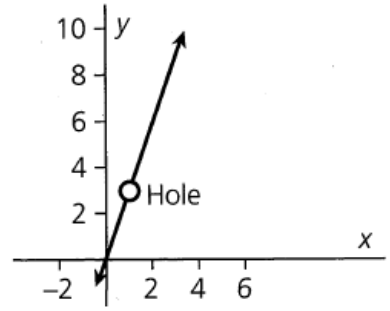
Section 3.5 – Continuity

When we talk about a function being ***continuous at a point***, we mean that the graph passes through the point without a break.
A graph that is ***not continuous at a point*** (sometimes referred to as being ***discontinuous at a point***) has a break of some type at the point. The following graphs illustrate these ideas.

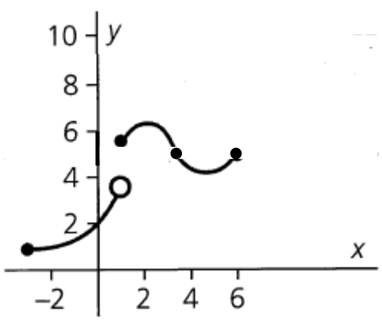
a. Continuous for all values of the domain



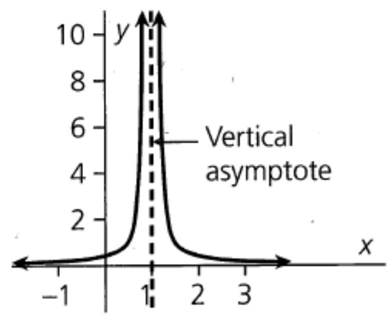
b. Discontinuous at $x = 1$ (removable discontinuity)



c. Discontinuous at $x = 1$ (jump discontinuity)



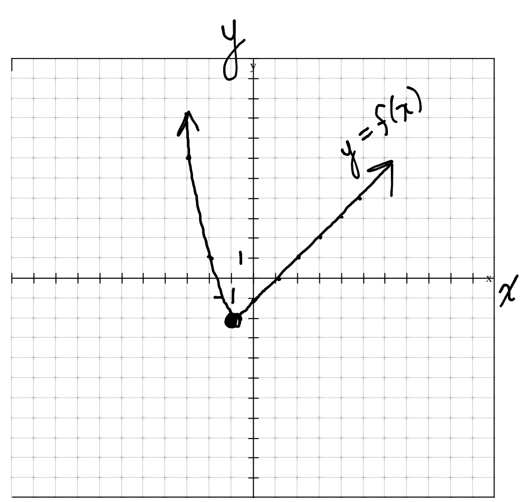
d. Discontinuous at $x = 1$ (infinite discontinuity)



A function f is ***continuous*** at $x = a$ if all three of the following conditions are satisfied:
- $f(a)$ is defined
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a} f(x) = f(a)$

Ex. 1. Given $f(x) = \begin{cases} x^2 - 3, & \text{if } x < -1 \\ x - 1, & \text{if } x \geq -1 \end{cases}$,

a) graph the function.



b) determine $\lim_{x \to -1} f(x)$.

$\lim_{x \to -1} f(x) = -2$

c) determine $f(-1)$.

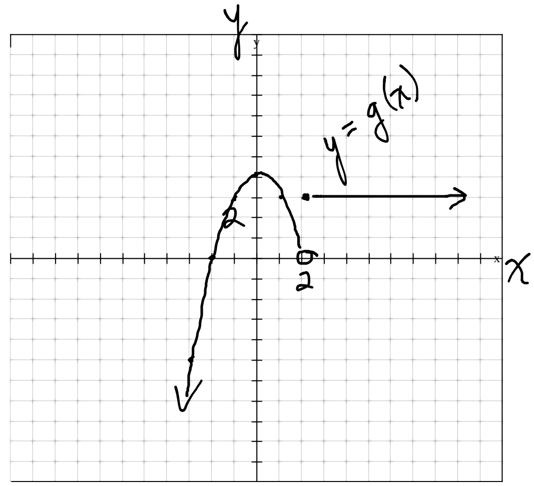
$f(-1) = -2$

d) is f continuous at $x = -1$? Explain.

yes, f is continuous at $x = -1$ since $\lim_{x \to -1} f(x) = f(-1)$

Ex. 2. Test the continuity of each of the following functions at $x = 2$.
If the function is not continuous at $x = 2$, give a reason why it is not continuous.
Illustrate graphically.

a) $g(x) = \begin{cases} 4 - x^2, & \text{if } x < 2 \\ 3, & \text{if } x \geq 2 \end{cases}$



- $g(2) = 3$
- Find $\lim_{x \to 2} g(x)$

L.S.L.

$= \lim_{x \to 2^-} (4 - x^2)$

$= 4 - (2)^2$

$= 0$

R.S.L.

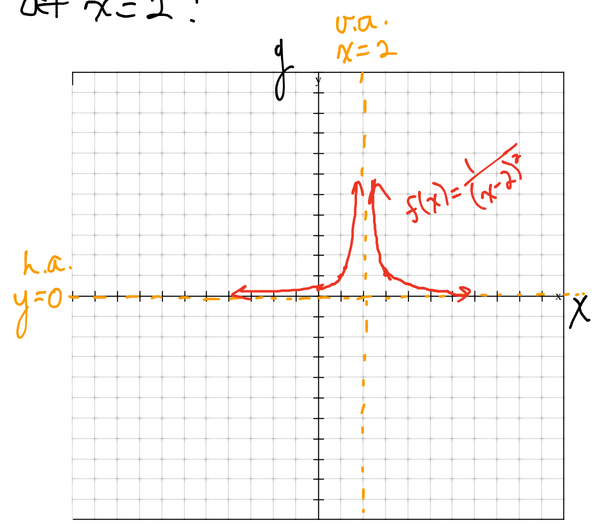
$= \lim_{x \to 2^+} g(x)$

$= \lim_{x \to 2^+} 3$

$= 3$

$\because \lim_{x \to 2^-} g(x) \neq \lim_{x \to 2^+} g(x)$, $\therefore \lim_{x \to 2} g(x)$ d.n.e & $g(x)$ is discontinous at $x = 2$!

b) $f(x) = \dfrac{1}{(x-2)^2}$

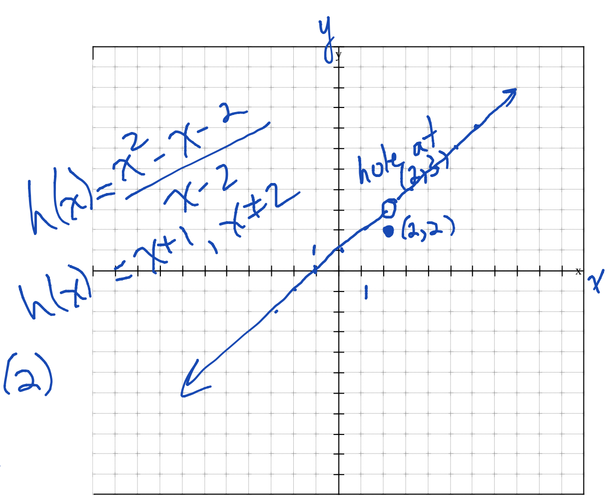


- $f(2)$ d.n.e

$\because f(2)$ is undefined,

$\therefore$ the function is discontinuous at $x = 2$.

c) $h(x) = \dfrac{x^2 - x - 2}{x - 2}$, $x \neq 2$ and $h(2) = 2$



- $h(2) = 2$
- Find $\lim_{x \to 2} h(x)$

$= \lim_{x \to 2} \dfrac{x^2 - x - 2}{x - 2} \quad \left(\dfrac{0}{0}\right)$

$= \lim_{x \to 2} \dfrac{(x-2)(x+1)}{(x-2)}$

$= \lim_{x \to 2} (x+1)$

$= 3$

$\because \lim_{x \to 2} h(x) \neq h(2)$

$\therefore$ The function is discontinuous at $x = 2$

HW: p. 110 #1, 4 to 8, 10 to 13

Review for Test: p. 115 #1 to 12, 13 to 15 (Evaluate using factoring or rationalizing techniques), 16 to 18; Day 6 Worksheet #1 to 3 even parts; p. 119 #4, 9, 11 to 17