Date: Feb.26/14 **Section 4.6 - THE POWER OF A FUNCTION RULE**

Recall:

Power Rule: If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$

Product Rule: If $y = f(x) \cdot g(x)$ then $\frac{dy}{dx} = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

Extended Product Rule: If $y = f(x) \cdot g(x) \cdot h(x)$ then

$$\frac{dy}{dx} = f'(x) \cdot g(x) \cdot h(x) + g'(x) \cdot f(x) \cdot h(x) + h'(x) \cdot f(x) \cdot g(x)$$

Ex. 1. Differentiate each of the following using the **Product Rules** and **Power Rule**.

a) $y = (3x^3 - 2x^2 - 1)^3$

Solution:

$$y = (3x^3 - 2x^2 - 1) \cdot (3x^3 - 2x^2 - 1) \cdot (3x^3 - 2x^2 - 1)$$

$$\frac{dy}{dx} = (9x^2 - 4x) \cdot (3x^3 - 2x^2 - 1)^2 + (9x^2 - 4x) \cdot (3x^3 - 2x^2 - 1)^2 + (9x^2 - 4x) \cdot (3x^3 - 2x^2 - 1)^2$$

$$\frac{dy}{dx} = 3(9x^2 - 4x)(3x^3 - 2x^2 - 1)^2$$

$$\therefore \frac{dy}{dx} = 3(3x^3 - 2x^2 - 1)^2 (9x^2 - 4x)$$

b) $y = [f(x)]^4$

Solution:

$$y = [f(x) \cdot f(x) \cdot f(x)] \cdot f(x)$$

$$\frac{dy}{dx} = \underbrace{\left[f'(x) \cdot [f(x)]^2 + f'(x) \cdot [f(x)]^2 + f'(x) \cdot [f(x)]^2\right]}_{} \cdot f(x) + f'(x) \cdot [f(x)]^3$$

$$\frac{dy}{dx} = 3f'(x) \cdot [f(x)]^2 \cdot f(x) + f'(x) \cdot [f(x)]^3$$

$$\frac{dy}{dx} = 3f'(x) \cdot [f(x)]^3 + f'(x) \cdot [f(x)]^3$$

$$\therefore \frac{dy}{dx} = 4[f(x)]^3 \cdot f'(x)$$

POWER OF A FUNCTION RULE: **If** $y = [f(x)]^n$ **then** $\frac{dy}{dx} = n[f(x)]^{n-1} \cdot f'(x)$

If $y = au^n$ **then** $\frac{dy}{dx} = nau^{n-1} \cdot \frac{du}{dx}$

Ex. 2. Differentiate each of the following using the **Power of a Function Rule**.

a) $y = [f(x)]^4$

$\frac{dy}{dx} = 4[f(x)]^3 \cdot f'(x)$

b) $y = (3x^3 - 2x^2 - 1)^3$

$\frac{dy}{dx} = 3(3x^3 - 2x^2 - 1)^2 (9x^2 - 4x)$

POWER OF A FUNCTION RULE

If $y = a[f(x)]^n$ then $\frac{dy}{dx} = na[f(x)]^{n-1} \cdot f'(x)$

Ex. 2. Use the **Power Rule** or **Power of a Function Rule** to find the derivative.

a) $f(x) = \left(\frac{3}{2x}\right)^2 - \frac{1}{x^3} + \frac{1}{x}$

$f(x) = \frac{9x^0}{4x^2} - \frac{1x^0}{1x^3} + \frac{1x^0}{1x^1}$

$f(x) = \frac{9}{4}x^{-2} - x^{-3} + x^{-1}$

$f'(x) = -\frac{9}{2}x^{-3} + 3x^{-4} - x^{-2}$

$\therefore f'(x) = -\frac{9}{2x^3} + \frac{3}{x^4} - \frac{1}{x^2}$

b) $y = (2x+1)^6$

$\frac{dy}{dx} = 6(2x+1)^5 \cdot 2$

$\therefore \frac{dy}{dx} = 12(2x+1)^5$

c) $f(x) = \left(-x^4 - \pi^4\right)^4$

constant

$f'(x) = 4(-x^4 - \pi^4)^3(-4x^3 + 0)$

$f'(x) = -16x^3(-x^4 - \pi^4)^3$

d) $y = \left[(3-x)^3 - x\right]^2$

$\frac{dy}{dx} = 2\left[(3-x)^3 - x\right]^1 \left[3(3-x)^2(-1) - 1\right]$

$= 2\left[(3-x)^3 - x\right]\left[-3(3-x)^2 - 1\right]$

$\therefore \frac{dy}{dx} = -2\left[(3-x)^3 - x\right]\left[3(3-x)^2 + 1\right]$

e) $y = \frac{1}{\sqrt{2x-1}}$

$y = (2x-1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{\cancel{2}}(2x-1)^{-\frac{3}{2}}(\cancel{2})$

$= -(2x-1)^{-\frac{3}{2}}$

$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{(2x-1)^3}}$

f) $g(x) = \frac{-3}{(x^2-4)^1}$

$g(x) = -3(x^2-4)^{-1}$

$g'(x) = 3(x^2-4)^{-2}(2x)$

$g'(x) = 6x(x^2-4)^{-2}$

$g'(x) = \frac{6x}{(x^2-4)^2}$

Ex. 3. Use the **Power Rule, Power of a Function Rule** and **Product Rule** to differentiate each function. Express your answer in ***simplified factored form***.

a) $s = t^4 \cdot (1-4t^2)^3$

$\frac{ds}{dt} = 4t^3 \cdot (1-4t^2)^3 + 3(1-4t^2)^2(-8t) \cdot t^4$

$= 4t^3(1-4t^2)^3 - 24t^5(1-4t^2)^2$

$= 4t^3(1-4t^2)^2\left[(1-4t^2) - 6t^2\right]$

$\therefore \frac{ds}{dt} = 4t^3(1-4t^2)^2(1-10t^2)$

b) $f(x)=\underbrace{(2x-1)^5}\cdot\underbrace{(2-3x)^3}$

$$f'(x)=\underbrace{5(2x-1)^4(2)}\cdot(2-3x)^3+\underbrace{3(2-3x)^2(-3)}\cdot(2x-1)^5$$
$$=10(2x-1)^4(2-3x)^3-9(2-3x)^2(2x-1)^5$$
$$=(2x-1)^4(2-3x)^2\left[10(2-3x)-9(2x-1)\right]$$
$$\therefore f'(x)=(2x-1)^4(2-3x)^2(29-48x)$$

c) $y=\dfrac{(3-x)^2}{(3x-2)^3}$

$$y=\underbrace{(3-x)^2}\cdot(3x-2)^{-3}$$
$$\frac{dy}{dx}=\underbrace{2(3-x)^1(-1)}\cdot(3x-2)^{-3}-\underbrace{3(3x-2)^{-4}(3)}\cdot(3-x)^2$$
$$=-2(3-x)(3x-2)^{-3}-9(3x-2)^{-4}(3-x)^2$$
$$=-(3-x)(3x-2)^{-4}\left[2(3x-2)+9(3-x)\right]$$
$$=-(3-x)(3x-2)^{-4}(-3x+23)$$
$$\therefore \frac{dy}{dx}=+(3-x)(3x-2)^{-4}(3x-23) \text{ or } \frac{dy}{dx}=\frac{(3-x)(3x-23)}{(3x-2)^4}$$

d) $y=\sqrt[3]{\dfrac{1+x^2}{1-2x^2}}\rightarrow y=\left(\dfrac{1+x^2}{1-2x^2}\right)^{\frac{1}{3}}\rightarrow y=\dfrac{(1+x^2)^{\frac{1}{3}}}{(1-2x^2)^{\frac{1}{3}}}$

$$y=(1+x^2)^{\frac{1}{3}}\cdot(1-2x^2)^{-\frac{1}{3}}$$
$$\frac{dy}{dx}=\underbrace{\frac{1}{3}(1+x^2)^{-\frac{2}{3}}(2x)}\cdot(1-2x^2)^{-\frac{1}{3}}-\underbrace{\frac{1}{3}(1-2x^2)^{-\frac{4}{3}}(-4x)}\cdot(1+x^2)^{\frac{1}{3}}$$
$$=\frac{2}{3}x(1+x^2)^{-\frac{2}{3}}(1-2x^2)^{-\frac{1}{3}}+\frac{4}{3}x(1-2x^2)^{-\frac{4}{3}}(1+x^2)^{\frac{1}{3}}$$
$$=\frac{2}{3}x(1+x^2)^{-\frac{2}{3}}(1-2x^2)^{-\frac{4}{3}}\left[(1-2x^2)+2(1+x^2)\right]$$
$$=\frac{2}{\cancel{3}}x(1+x^2)^{-\frac{2}{3}}(1-2x^2)^{-\frac{4}{3}}(\cancel{3})$$
$$\therefore \frac{dy}{dx}=2x(1+x^2)^{-\frac{2}{3}}(1-2x^2)^{-\frac{4}{3}} \text{ or } \frac{dy}{dx}=\frac{2x}{(1+x^2)^{2/3}(1-2x^2)^{4/3}}$$
$$\text{or } \frac{dy}{dx}=\frac{2x}{\sqrt[3]{(1+x^2)^2}\cdot\sqrt[3]{(1-2x^2)^4}}$$

HW: p. 158 #2 to 4 all parts

Date: Feb. 27/14

Section 4.6 – THE CHAIN RULE

Recall:

If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$

If $y = a[f(x)]^n$ then $\frac{dy}{dx} = na[f(x)]^{n-1} \cdot f'(x)$

If $y = au^n$ then $\frac{dy}{dx} = nau^{n-1} \cdot \frac{du}{dx}$

Ex. 1. Find the rate of change of $f(t) = \left(\frac{\pi - t}{6\pi + t}\right)^{\frac{1}{3}}$ at $t = 2\pi$.

$$f(t) = \left(\frac{\pi - t}{6\pi + t}\right)^{\frac{1}{3}}$$

$$f'(t) = \frac{1}{3}\left(\frac{\pi - t}{6\pi + t}\right)^{-\frac{2}{3}} \cdot \frac{-1(6\pi + t) - 1(\pi - t)}{(6\pi + t)^2}$$

$$f'(2\pi) = \frac{1}{3}\left(\frac{-\pi}{8\pi}\right)^{-\frac{2}{3}} \cdot \frac{-8\pi + \pi}{(8\pi)^2}$$

$$= \frac{1}{3}\left(-\frac{1}{8}\right)^{-\frac{2}{3}} \cdot \frac{-7\pi}{64\pi^2}$$

$$= \frac{1}{3}(-8)^{\frac{2}{3}} \cdot \frac{-7}{64\pi}$$

$$= \frac{1}{3} \times \frac{\cancel{4}^{1}}{1} \times \frac{-7}{_{16}\cancel{64}\pi}$$

$$= \frac{-7}{48\pi}$$

$\therefore f'(2\pi) = \frac{-7}{48\pi}$

Ex. 2. The radius of a circular oil slick on a body of water is increasing at a $\overbrace{\text{rate of 2 m/min}}^{\frac{dr}{dt}}$.
Find the ***rate of change of area*** of the oil slick when its radius is 50 m.

Given $\frac{dr}{dt} = 2\text{ m/min}$

Find $\frac{dA}{dt}$ when $r = 50\text{ m}$

$A = \pi r^2$

differentiate w.r.t. "t"

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

if $r = 50$; $\frac{dr}{dt} = 2$

$\frac{dA}{dt} = 2\pi(50)(2)$
$= 200\pi$

$\therefore$ when the radius is 50m the area is changing at an exact rate of 200π m²/min or an approximate rate of 628.3 m²/min.

THE CHAIN RULE IN LEIBNIZ NOTATION

If y is a function of u and u is a function of x, then $$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$	If y is a function of u and u is a function of v and v is a function of x, then $$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$$

Ex. 3. Use the ***chain rule*** to find $\frac{dy}{dx}$ at $x=1$ if $y=2u^3+3u^2$ and $u=x+\frac{1}{\sqrt{x}}$.

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=(6u^2+6u)\left[1-\frac{1}{2(\sqrt{x})^3}\right]$$

at $x=1$, $u=2$

$$\frac{dy}{dx}=36\left(1-\frac{1}{2}\right)$$
$$=18$$

$\therefore \frac{dy}{dx}=18$ at $x=1$

$$y=2u^3+3u^2$$
$$\frac{dy}{du}=6u^2+6u$$

at $x=1$

$$u=(1)+\frac{1}{\sqrt{1}}$$
$$=2$$

$$u=x+x^{-\frac{1}{2}}$$
$$\frac{du}{dx}=1-\frac{1}{2}x^{-\frac{3}{2}}$$
$$=1-\frac{1}{2(\sqrt{x})^3}$$

Ex. 4. Use the ***chain rule*** to find $\frac{dy}{dx}$ at $x=4$ if $y=\frac{12}{(1+2u)^3}$, $u=1-\frac{2}{v^2}$ and $v=2x-3\sqrt{x}$.

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$$

$$=\frac{-72}{(1+2u)^4}\cdot\frac{4}{v^3}\cdot\left(2-\frac{3}{2\sqrt{x}}\right)$$

at $x=4$, $v=2$, $u=\frac{1}{2}$

$$\frac{dy}{dx}=\frac{\cancel{-72}\ 9}{\cancel{16}\ 4}\cdot\frac{\cancel{4}\ 1}{\cancel{8}\ 2}\cdot\left(2-\frac{3}{4}\right)$$

$$=-\frac{9}{4}\times\frac{5}{4}$$

$$=-\frac{45}{16}$$

$\therefore \frac{dy}{dx}=-\frac{45}{16}$ at $x=4$

$$y=12(1+2u)^{-3}$$
$$\frac{dy}{du}=-36(1+2u)^{-4}(2)$$
$$=\frac{-72}{(1+2u)^4}$$

$$v=2x-3x^{\frac{1}{2}}$$
$$\frac{dv}{dx}=2-\frac{3}{2}x^{-\frac{1}{2}}$$
$$=2-\frac{3}{2\sqrt{x}}$$

$$u=1-2v^{-2}$$
$$\frac{du}{dv}=4v^{-3}$$
$$=\frac{4}{v^3}$$

If $x=4$

$v=8-6$

$=2$

If $v=2$

$$u=1-\frac{2}{4}$$
$$=\frac{1}{2}$$

THE CHAIN RULE FOR A COMPOSITE FUNCTION

If $y = f(g(x))$ then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

"The derivative of a composite function is the derivative of the outer function with respect to the inner function times the derivative of the inner function with respect to x."

Ex. 5. Let $y = g(h(x))$ where $h(x) = \frac{x^2}{x+2}$. If $g'\left(\frac{9}{5}\right) = -2$, find $\frac{dy}{dx}$ when $x = 3$.

$y = g(h(x))$

$\frac{dy}{dx} = g'(h(x)) \cdot h'(x)$

at $x = 3$

$\frac{dy}{dx} = g'(h(3)) \cdot h'(3)$

$= \underbrace{g'\left(\frac{9}{5}\right)} \cdot \frac{21}{25}$

$= \frac{-2}{1} \times \frac{21}{25}$

$= \frac{-42}{25}$

$\therefore \frac{dy}{dx} = \frac{-42}{25}$ when $x = 3$.

$h(3) = \frac{(3)^2}{(3)+2}$

$= \frac{9}{5}$

$h(x) = \frac{x^2}{x+2}$

$h'(x) = \frac{2x(x+2) - 1(x^2)}{(x+2)^2}$

$h'(3) = \frac{(6)(5) - 9}{25}$

$= \frac{21}{25}$

HW: p. 159 #5a, 6 to 10, 12 to 15; Day 8: REVIEW for TEST: p. 163 #2 to 20; p. 166 #3 to 11