Date:____________ **UNIT 3: FUNCTIONS & TRANSFORMATIONS**

3.1 Relations, Domain, Range and Functions

A. Definitions

Relation: *A* ***relation*** *is an identified relationship between two variables that may be expressed as a set of* ***ordered pairs****, a* ***table of values****, a* ***graph****, or an* ***equation****.*

Domain: *The* ***domain*****, D***, is the set of all first elements (x-values) of a relation.*

Range: *The* ***range*****, R***, is the set of all second elements (y-values) of a relation.*

Function: *A* ***function*** *is a relation in which each first element (x-value) corresponds to exactly one second element (y-value).*

Vertical Line Test: *If any* ***vertical line*** *intersects the graph of a relation more than once, then the relation is not a function.*

B. Examples

1. State the domain of each of the following relations.

a) $y = \dfrac{x-1}{x^2-3x-4}$

$y = \dfrac{x-1}{(x-4)(x+1)}$, $x = -1, 4$

$D = \{x \in \mathbb{R} \mid x \neq -1, 4\}$

"The Domain equals the set of x-values that are elements of the real numbers such that x is not equal to −1 or 4"

b) $s = \sqrt{8-2t}$

$8 - 2t \geq 0$

$-2t \geq -8$

$t \leq 4$

$D = \{t \in \mathbb{R} \mid t \leq 4\}$

"is the set of"

"is an element of..."

"such that"

c) $y = \dfrac{1}{\sqrt{x^2-9}}$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

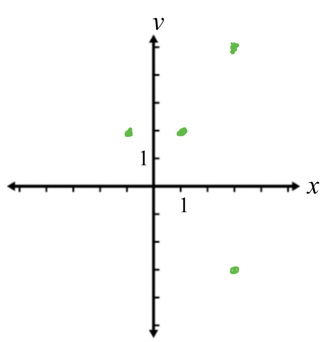
zeros are $x = -3$ & $x = 3$

−3 3 x

$D = \{x \in \mathbb{R} \mid x < -3 \text{ or } x > 3\}$

2. State the domain and range. Determine if the relation is a function. Explain.

a) $\{(1,2), (3,5), (3,-3), (-1,2)\}$



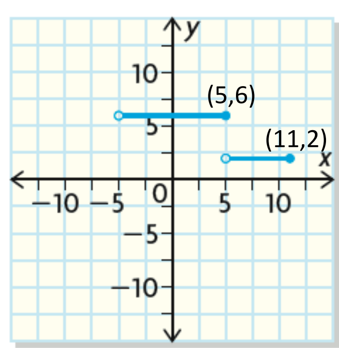
i) $D =$ $\{-1, 1, 3\}$

ii) $R =$ $\{-3, 2, 5\}$

iii) *Function?* No

The x-value of 3 has more than one y-value.

b)



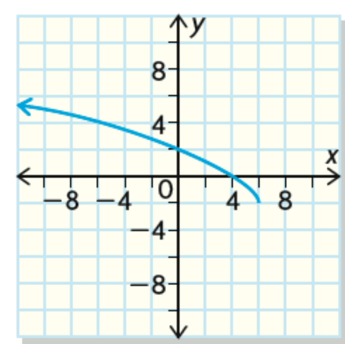
i) $D =$ $\{x \in \mathbb{R} \mid -5 < x \leq 11\}$

ii) $R =$ $\{2, 6\}$

iii) *Function?* Yes

The graph passes the vertical line test.

c)



i) $D =$ $\{x \in \mathbb{R} \mid x \leq 6\}$

ii) $R =$ $\{y \in \mathbb{R} \mid y \geq -2\}$

iii) *Function?* Yes

The graph passes the vertical line test.

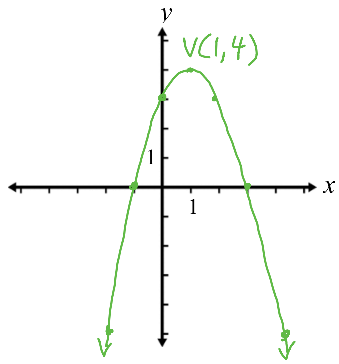
3. Graph, and state the domain and range. Determine if the relation is a function. Explain.

a) $y = -x^2 - 2x + 3$

$y = -(x^2+2x+1-1)+3$

$y = -(x+1)^2+4$

- $V(-1,4)$
- opens down
- congruent to $y=x^2$



i) $D =$ $\{x \in \mathbb{R}\}$

ii) $R =$ $\{y \in \mathbb{R} \mid y \leq 4\}$

iii) *Function?* Yes

The graph passes the vertical line test.

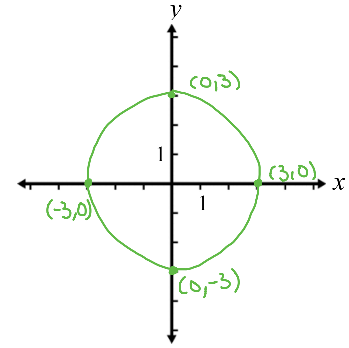
b) $x^2 + y^2 = 9$

Circle

Centre $(0,0)$

$r^2=9$

$r=3$



i) $D =$ $\{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$

ii) $R =$ $\{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

iii) *Function?* No

The graph fails the vertical line test.

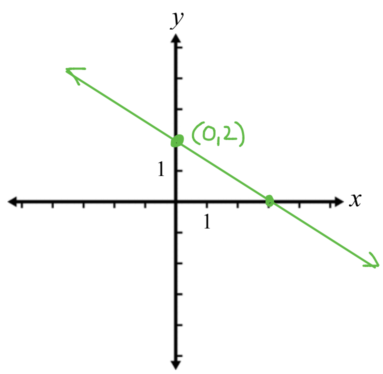
c) $2x + 3y = 6$

$3y = -2x+6$

$y = -\frac{2}{3}x+2$

slope: $m = -\frac{2}{3}$

y-int.: $b = 2$.



i) $D =$ $\{x \in \mathbb{R}\}$

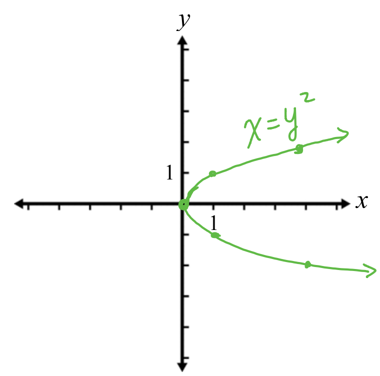
ii) $R =$ $\{y \in \mathbb{R}\}$

iii) *Function?* Yes

The graph passes the vertical line test.

d) $x = y^2$

x	y
4	-2
1	-1
0	0
1	1
4	2



i) $D =$ $\{x \in \mathbb{R} \mid x \geq 0\}$

ii) $R =$ $\{y \in \mathbb{R}\}$

iii) *Function?* No

The graph fails the vertical line test.

HW. Exercise 3.1

Date:____________ **3.2 Function Notation**

A. Definition

Symbols such as $\boldsymbol{f(x)}$, *read as "$\boldsymbol{f}$ at $\boldsymbol{x}$" or "$\boldsymbol{f}$ of $\boldsymbol{x}$",* $\boldsymbol{g(x)}$ *and* $\boldsymbol{h(x)}$, *are called* ***function notation***.
$\boldsymbol{f(x)}$ *represents the value of* $\boldsymbol{y}$ *for a given value of* $\boldsymbol{x}$, *so* $\boldsymbol{y = f(x)}$.

B. Examples

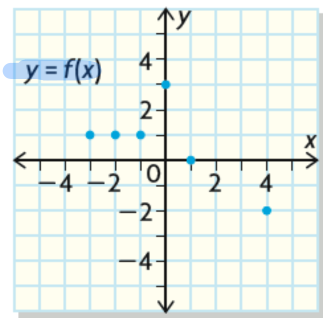
1. Rewrite the following equations in ***function notation***.

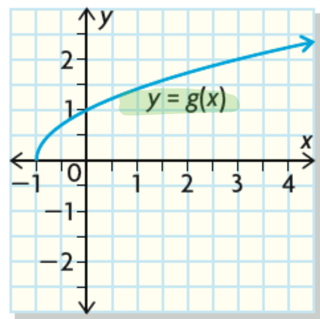
a) $y = -3x + 2$
$f(x) = -3x + 2$

b) $y = -2x^2 - 5x + 2$
$g(x) = -2x^2 - 5x + 2$

c) $h = -5t^2 + 30t + 4$
$h(t) = -5t^2 + 30t + 4$

2. Using the graphs of $y = f(x)$ and $y = g(x)$ determine value(s) for each of the following.





a) $D_f = \{-3, -2, -1, 0, 1, 4\}$

$R_f = \{-2, 0, 1, 3\}$

b) $D_g = \{x \in \mathbb{R} \mid x \geq 1\}$

$R_g = \{y \in \mathbb{R} \mid y \geq 0\}$

c) $f(0)$
$= 3$

d) $g(-1)$
$= 0$

e) $g(3) - f(4)$
$= 2 - (-2)$
$= 4$

f) x if $g(x) = 1$
$x = 0$

g) x if $f(x) = 1$
$x = -3$ or $x = -2$ or $x = -1$

3. If $f(x) = -3x + 2$, find

a) $f(-1)$
$= -3(-1) + 2$
$= 5$

b) $f(0)$
$= -3(0) + 2$
$= 2$

c) $f(2)$
$= -3(2) + 2$
$= -4$

d) $f\left(\frac{1}{6}\right) - f\left(-\frac{1}{9}\right)$
$= -3\left(\frac{1}{6}\right) + 2 - \left[-3\left(-\frac{1}{9}\right) + 2\right]$
$= -\frac{1}{2} + 2 - \frac{1}{3} - 2$
$= -\frac{3}{6} - \frac{2}{6}$
$= -\frac{5}{6}$

e) $f(-3a)$
$= -3(-3a) + 2$
$= 9a + 2$

f) $f(x - 3)$
$= -3(x - 3) + 2$
$= -3x + 9 + 2$
$= -3x + 11$

4. If $g(x) = -2x^2 - 5x + 2$, find

a) $g(-1)$
$= -2(-1)^2 - 5(-1) + 2$
$= -2 + 5 + 2$
$= 5$

b) $g(3)$
$= -2(3)^2 - 5(3) + 2$
$= -18 - 15 + 2$
$= -31$

c) $g\left(\frac{1}{2}\right)$
$= -2\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + \frac{2}{1}$
$= \cancel{-2}^{-1}\left(\frac{1}{\cancel{4}_{2}}\right) - \frac{5}{2} + \frac{4}{2}$
$= -\frac{1}{2} - \frac{5}{2} + \frac{4}{2}$
$= -\frac{2}{2}$
$= -1$

d) $g(3x)$
$= -2(3x)^2 - 5(3x) + 2$
$= -2(9x^2) - 15x + 2$
$= -18x^2 - 15x + 2$

e) $g(a + 3)$
$= -2(a+3)(a+3) - 5(a+3) + 2$
$= -2(a^2 + 6a + 9) - 5(a+3) + 2$
$= -2a^2 - 12a - 18 - 5a - 15 + 2$
$= -2a^2 - 17a - 31$

5. If $f(x)=5x+8$, $g(x)=x^2+3x$ and $h(x)=\sqrt{4x+1}$, find the value(s) of x, if

a) $f(x)=-7$

$5x+8=-7$

$5x=-15$

$x=-3$

b) $g(x)=18$

$x^2+3x=18$

$x^2+3x-18=0$

$(x+6)(x-3)=0$

$\therefore x=-6$ or $x=3$

c) $f(x-1)=g(2x+1)$

$5(x-1)+8=(2x+1)(2x+1)+3(2x+1)$

$5x-5+8=4x^2+4x+1+6x+3$

$5x+3=4x^2+10x+4$

$0=4x^2+5x+1$

$0=(4x+1)(x+1)$

$\therefore x=-1$ or $x=-\frac{1}{4}$

d) $h(x)=3\sqrt{5}$

$\sqrt{4x+1}=3\sqrt{5}$

$(\sqrt{4x+1})^2=(3\sqrt{5})^2$

$4x+1=9(5)$

$4x+1=45$

$4x=44$

$x=11$

e) $h(x^2-x)=0$

$\sqrt{4(x^2-x)+1}=0$

$(\sqrt{4x^2-4x+1})^2=(0)^2$

$4x^2-4x+1=0$

$(2x-1)(2x-1)=0$

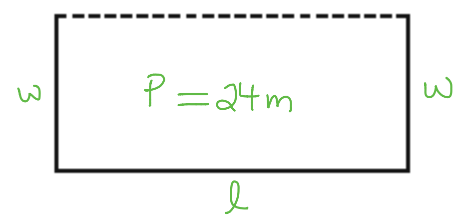
$\therefore x=\frac{1}{2}$ or $x=\frac{1}{2}$

6. Sarah has 24 m of fencing to enclose a rectangular garden against the back of her house.

a) Express the area, A, in m^2 of the garden as a function of its width, w, in m.

b) Determine the domain and range of the area function.

c) Determine the dimensions that give the maximum area.



* Find l in terms of w:

$l+2w=24$

$l=24-2w$

a) $A=l^{*}\times w$

$A=w(24-2w)$

$A=-2w^2+24w$

$\therefore A(w)=-2w^2+24w$

b) For the domain, $A(w)>0$

$-2w^2+24w>0$

$2w^2-24w<0$

$2w(w-12)<0$

$0 \quad 12 \quad w$

$\therefore D=\{w\in\mathbb{R}\mid 0<w<12\}$

$R=\{A(w)\in\mathbb{R}\mid 0<A(w)\leq 72\}$

For Range:

$A(w)=-2(w^2-12w+36-36)$

$A(w)=-2(w-6)^2+72$

$V(6,72)$

c) For the maximum area $w=6$m and $l=12$m.

HW. Exercise 3.2