

# MPM1DI

## Unit 4: Algebra

Name: \_\_\_\_\_



### 4.1 Communicate With Algebra

#### Terms and Degrees of Terms:

A **term** is an expression formed by the **product** of **numbers** and/or **variables** . In the term  $-3x^2y$ , \_\_\_\_\_ is the \_\_\_\_\_, and \_\_\_\_\_ is the \_\_\_\_\_.

The **degree of a term** is the sum of the exponents on the variable.

**Ex. 1.** Identify the coefficient and the variable part of each term and state the degree of the term.

Term	Coefficient	Variable	Degree
$x^2$			
$3y^4$			
$0.7u$			
$-2a^2b$			
$\frac{2}{3}xy$			
$-5$			

#### Polynomials and Degrees of Polynomials:

**One term** is called a \_\_\_\_\_ eg.  $2m^2n^3$

**Two terms** connected by + or – signs is called a \_\_\_\_\_ eg.  $2x^2 - 5x$

**Three terms** connected by + or – signs is called a \_\_\_\_\_ eg.

**Many terms** connected by + or – signs is called a \_\_\_\_\_ eg.

The **degree of a polynomial** is the degree of the highest degree term.

**Ex. 2.** Classify each polynomial by the number of terms it has and state the degree of the polynomial.

Polynomial	Type of Polynomial	Degree of Polynomial
$3x^2y^2 + 2x^2y$		
$2m^2n^3$		
$3y^3 + 0.2y - 8$		
$a - 2b + c - 3$		
$-0.5$		

### Using an Algebraic Model to Solve a Problem:

**Ex. 3.** A small pizza costs \$7.00 plus \$1.50 per topping.

**a)** Define the variable and write an expression that describes the total cost of a small pizza in terms of the variable.

**b)** Determine the cost of a small pizza with 5 toppings.

**Ex. 4.** Cheryl works part-time as a ski instructor. She earns \$125 for the season, plus \$20 for each children's lesson and \$30 for each adult lesson that she gives.

**a)** Define the variables and write an expression that describes Cheryl's total earnings in terms of the variables.

**b)** One winter, Cheryl gave eight children's lessons and six adult lessons. What were her total earnings?

## 4.1 Communicate With Algebra Worksheet

1. Identify the coefficient and the variable part of each term.

a)  $2y$

b)  $-3x$

c)  $mn$

d)  $\frac{1}{2}x^2$

e)  $-w^2$

f)  $-0.4gh^3$

2.  $7x^2 + 3xy + 4y^2$  is a:

A. monomial

B. binomial

C. trinomial

D. term

3. Classify each polynomial by the number of terms.

a)  $-2x$

b)  $6y^2 + 2y - 1$

c)  $a - \frac{1}{2}b$

d)  $3u^2 - uv + 2v^2$

e)  $3k^3 - \frac{1}{2}k$

f)  $m + 0.2n - 0.3 + mn$

4. The degree of  $4u - 5u^2 + 9$  is:

A. 1

B. 2

C. 3

D. 0

5. State the degree of each term.

a)  $5x^2$

b)  $-6y$

c)  $-3$

d)  $u^2v^4$

e)  $\frac{1}{3}x^2y^3$

f)  $0.2a^2b$

6. State the degree of each polynomial.

a)  $3x - 4$

b)  $y^2 + 3y - 1$

c)  $m - 2m^3$

d)  $a^3b^2 - 8a^2b^5$

e)  $2x^2y^4 - \frac{2}{5}xy^3$

7. In a TV trivia show, a contestant receives 500 points for a correct answer and loses 200 points for an incorrect answer. Let  $c$  represent the number of correct answers and  $i$  represent the number of incorrect answers. Which expression describes a contestant's total points?

A.  $500c + 200i$

B.  $500c - 200i$

C.  $500i + 200c$

D.  $500i - 200c$

8. A hockey team earns 2 points for a win and 1 point for a tie. Let  $w$  represent the number of wins and  $t$  represent the number of ties. Which expression can be used to describe the team's total points? Is there more than one correct answer?

A.  $2w + 1$

B.  $w + t$

C.  $2w + 1t$

D.  $2w + t$

9. Substitute the given values and evaluate each expression.

a)  $3x + 5$        $x = 2$

b)  $4y + 4$        $y = -2$

c)  $a^2 + 2b - 7$        $a = 4, b = 1$

d)  $2m^2 - 3n + 8$        $m = -2, n = 5$

10. Shaylee has a summer job at a fitness club. She earns a \$5 bonus for each student membership and a \$7 bonus for each adult membership she sells.
- Define your variables and write a polynomial expression that describes Shaylee's total bonus.
  - How much with Shaylee's bonus be if she sells 12 student memberships and 10 adult memberships?
11. An arena charges \$25 for gold seats, \$18 for red seats and \$15 for blue seats.
- Define your variables and write an expression that describes the total earnings from seat sales.
  - How much with the arena earn if it sells 100 gold seats, 200 blue seats and 250 red seats?
12. On a multiple choice test, you earn 2 points for each correct answer and lose one point for each incorrect answer.
- Define your variables and write an expression for a student's total score.
  - Nolan answered 15 questions correctly and 3 incorrectly. Find Nolan's total score.
13. Aiden is training for a triathlon, where athletes swim, cycle, and run. During his training program, he has found that he can swim at 1.2 km/h, cycle at 25 km/h and run at 10 km/h. To estimate his time for an upcoming race, Aiden rearranges the formula  $distance = speed \times time$  to find that:

$$time = \frac{distance}{speed}$$

- Choose a variable to represent the distance travelled for each part of the race. For example, choose  $s$  for the swim.
- Copy and complete the table. The first row is done for you.

Part of the Race	Speed (km/h)	Distance (km)	Time (h)
swim	1.2	$s$	$\frac{s}{1.2}$
cycle			
run			

- Write a trinomial to model Aiden's total time.
- A triathlon is advertised in Kingston. Participants have to swim 1.5 km, cycle 40 km, and run 10 km. Using your expression from part c), calculate how long it will take Aiden to finish the race.
- Is your answer a reasonable estimate of Aiden's triathlon time? Explain.

### Answers

1. **A.** coefficient: 2, variable:  $y$ , **B.** coefficient: -3, variable:  $x$ , **C.** coefficient: 1, variable:  $mn$ , **D.** coefficient:  $\frac{1}{2}$ , variable:  $x^2$ , **E.** coefficient: -1, variable:  $w^2$ , **F.** coefficient: -0.4, variable:  $gh^3$ . **2. C. 3. A.** monomial, **B.** trinomial, **C.** binomial, **D.** trinomial, **E.** binomial, **F.** four-term polynomial. **4. B. 5. A.** 2, **B.** 1, **C.** 0, **D.** 6, **E.** 5, **F.** 3. **6. A.** 1, **B.** 2, **C.** 3, **D.** 7, **E.** 6. **7. B. 8. C & D.** Both are correct. **9. A.** 11, **B.** -4, **C.** 11, **D.** 1. **10. A.**  $5s+7a$  **B.** \$130. **11. A.**  $25g + 18r + 15b$ , **B.** \$10 000
- 12. A.**  $2c - w$ , **B.** 27. **13. C.**  $\frac{s}{1.2} + \frac{c}{25} + \frac{r}{10}$ , **D.** 3.85 h, **E.** This is a reasonable time for a triathlon considering the length.

## 4.2 Adding and Subtracting Polynomials

**Like Terms** have the exact same variable parts. The exponents on all variables must be identical for terms to be like.

**Ex. 1.** Circle the like terms in each group and identify their coefficients.

a)  $2x, -3x^2, -x$

b)  $4m^2, 2m, -3m^2$

c)  $ab^2, 4a^2b, -2b^2a$

**Rule:** When simplifying polynomial expressions without brackets, only collect (add or subtract) “like” terms.

**Ex. 2.** Simplify.

a)  $2x - 3 + 5x + 1$

b)  $-x - x - x + x$

c)  $\frac{3}{5}a - \frac{1}{2}b - \frac{1}{5}a + \frac{3}{4}b$

d)  $-3u + 2 - u^2 - 5 + 3u + 2u^2$

**Ex. 3.** Simplify to a single power by using the exponent laws.

a)  $2^{3x} \times 2^{4x} \div 2^{-x}$

b)  $3^{2ab} \div 3^{3a^2} \cdot 3^{-4ba}$

**Rule:** When adding or subtracting polynomials in brackets multiply every term in the brackets by +1 or -1.

**Ex. 4.** Simplify.

a)  $(-5p^2 - 2p) + (3p^2 - 5p)$

b)  $(3x^2y + 5xy - 9) + (2x^2y - 4xy + 4)$

c)  $(x + y - z) - (x - y + z)$

d)  $(5y^2 - 5y + 1) - (-2y^2 - 3)$

e)  $(3x^2 - 5x + 1) - (2x^2 - 3) + (x^2 - 3x)$

f)  $\left(\frac{3}{4}x - \frac{2}{3}y\right) - \left(-\frac{1}{2}x + \frac{5}{7}y\right)$

**Ex. 5.** Simplify to a single power by using the exponent laws and evaluate if possible.

a)  $\frac{2^{(3x-8)}}{2^{(5x-4)}}$

b)  $3^{(7y^2-8y)} \times 3^{(-3y^2-5y)} \div 3^{(4y^2-13y)}$



## 4.2 Adding and Subtracting Polynomials Worksheet

1. Circle the like terms in each group and identify their coefficients.

a)  $3x, 4y, -x$

b)  $6m, -1.5m, 4n, 3m^2$

c)  $-2fg, 3f^2g, 2fg^2, gf$

d)  $3y, -2.1y^3, -0.8y^3, 0.5y^2$

e)  $-\frac{1}{2}x^2y^2, \frac{2}{3}x^2y, 1\frac{1}{4}y^2x^2, \frac{3}{5}xy^2$

2. Write each polynomial in descending powers of the variable.

a)  $2x - 1 + x^2$

b)  $-4 - 6y^4 + 2y^2$

c)  $1 + b - b^3 - b^2$

3. Simplify each of the following by adding and subtracting "like" terms.

a)  $3x - 2 + 6x + 4$

b)  $9c - 18 + 6c - 4$

c)  $k - 4k - 6 - 7k + 2$

d)  $3x - 2y - 7x + 5y$

e)  $x^2 - 13 + 5x^2 + 6$

f)  $8t - 9t^2 + 3t^2 - t$

g)  $-8a + 2a^2 - 3a + 3a^2 - 7 + 2a - 6a^2$

h)  $2k^2 - 5k + 1 + k^2 + 3k - 5$

i)  $3 - 8x^3 - 2x^2 + 7 - 5x^2 + 2x^3$

j)  $3xy + 5yz - 2xyz + 6xy - xyz$

k)  $7m - 4mn - 7m + 2mn$

l)  $-4x^2 + 3xy + yx + y^2 + 7x^2 - 4y^2$

m)  $-1 - y^2 - 5y - y^2 + 6y - 4y^2$

4. Simplify.

a)  $3.6 - 0.18x - (-2.31x) + (-4.3)$

b)  $\frac{3}{4}a - 0.\bar{6}b - 0.25a - \frac{4}{3}b$

c)  $\frac{4}{3}k^2 + \frac{2}{5}k - 3k^2 + \frac{1}{2}k$

5. Simplify.

a)  $(-2x + 3) + (3x - 4)$

b)  $(3x + 2) - (5x + 2)$

c)  $(4a - 9b) - (-2a + 3b)$

d)  $(0.35x + 1.7) + (3.4 - 0.82x)$

e)  $\left(\frac{2}{5}m - \frac{3}{8}n\right) - \left(\frac{3}{5}m - \frac{5}{8}n\right)$

f)  $\left(-\frac{3}{4}s + \frac{1}{2}t\right) + \left(\frac{4}{5}s - \frac{5}{6}t\right)$

g)  $(5y^2 - 2y + 6) - (2y^2 + 3y - 1)$

h)  $(-4c + 6c^4 + c^3) + (3c^4 - 2c^3 + 5c^2) - (10c^4 + c)$

i)  $(3x^2 - 4xy + 6y^2) - (6x^2 - 8yx - 3y^2)$

j)  $(9x^2y^2 - 3xy + 1) - (4x^2y^2 - 7) + (5xy - 8)$

6. Simplify to a single power by using the exponent laws and evaluate if possible.

a)  $\frac{(5^{x+1})(5^{-4x})}{5^{-3x-1}}$

b)  $(-2)^{(3a^2-3a+1)} \div (-2)^{(5a^2-5a+7)} \times (-2)^{(2a^2-2a+3)}$

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**Answers**

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1. a)  $3x, -1x$  b)  $6m, -1.5m$  c)  $-2fg, 1fg$  d)  $-2.1y^3, -0.8y^3$  e)  $-1/2x^2y^2, 5/4x^2y^2$  2. a)  $x^2 + 2x - 1$  b)  $-6y^4 + 2y^2 - 4$  c)  $-b^3 - b^2 + b + 1$

3. a)  $9x + 2$  b)  $15c - 22$  c)  $-10k - 4$  d)  $-4x + 3y$  e)  $6x^2 - 7$  f)  $-6t^2 + 7t$  g)  $-a^2 - 9a - 7$  h)  $3k^2 - 2k - 4$  i)  $-6x^3 - 7x^2 + 10$

j)  $-3xyz + 9xy + 5yz$  k)  $-2mn$  l)  $3x^2 + 4xy - 3y^2$  m)  $-6y^2 + y - 1$  4. a)  $2.13x - 0.7$  b)  $\frac{1}{2}a - 2b$  c)  $-\frac{5}{3}k^2 + \frac{9}{10}k$

5. a)  $x - 1$  b)  $-2x$  c)  $6a - 12b$  d)  $-0.47x + 5.1$  e)  $-\frac{1}{5}m + \frac{1}{4}n$  f)  $\frac{1}{20}s - \frac{1}{3}t$  g)  $3y^2 - 5y + 7$  h)  $-c^4 - c^3 + 5c^2 - 5c$

i)  $-3x^2 + 4xy + 9y^2$  j)  $5x^2y^2 + 2xy$  6. a)  $5^2 = 25$  b)  $(-2)^{-3} = -\frac{1}{8}$

### 4.3 Multiplying a Polynomial by a Monomial

Algebraic expressions can be simplified using the **Distributive Property**. The term in front of the brackets is **“shared”** with every term inside the brackets through multiplication.

**Ex. 1.** Simplify the following expressions by first multiplying, then collecting “like” terms when possible.

a)  $2(3x - 1)$

b)  $-5(3x^2 - 2y + 4)$

c)  $(3 - 6w)(-2)$

d)  $\frac{5}{7}\left(\frac{3}{4}x - \frac{1}{2}y + \frac{2}{3}\right)$

e)  $\frac{1}{2}(2w - 6) - \frac{2}{3}(9w - 6)$

f)  $18 - 2(x - 3) + 4x$

g)  $3[2 + 5(2k - 1)]$



### 4.3 Multiplying a Polynomial by a Monomial Worksheet

1. Simplify the following by expanding.

a)  $5(3h + 2)$

b)  $-6(4x - 3)$

c)  $(3x - 2y)(4)$

d)  $-5(-2a + 3b - 8c)$

2. Simplify the following by expanding and adding "like" terms.

a)  $-2(5y - 3) + 3(2y - 7)$

b)  $5(2x - 3y) - 4(3x + 6y)$

c)  $2(5 - 2m + 3m^2) + (m^2 - 4)$

d)  $3(a^2 - 2a - 1) - (3a^2 - a - 4)$

e)  $4(x^2 + 4xy + 4y^2) - 3(9x^2 - 12xy + 4y^2)$

f)  $-5(3p^2q^2 - 4pq) + 3(2pq - 3)$

g)  $3(2x - 4) - 2(x^2 + 3x - 5) + 3x^2$

h)  $5(3x^2 - y^2) - 2(3xy) - 3(x^2 - 2xy + y^2)$

i)  $-4(2m - 3n + 1) + 6(-m + 5n - 1) - 3(4m - n)$

j)  $4 - 3(5 - 4x) + 2(-5x)$

k)  $3[x + 2(x - 4)]$

l)  $2(3k - 1) - 4[2k - (2 + 3k)]$

3. Expand.

a)  $\frac{1}{4}(12 + 4x)$

b)  $-\frac{1}{3}(15x^2 - 12x - 3)$

c)  $-\frac{3}{2}\left(\frac{7}{3}r - \frac{5}{6}s\right)$

4. Simplify.

a)  $-0.2(0.3a + 0.5b - 10)$

b)  $-3.15 + 3(1.3x - 0.09) + (1.85 - 2.8x)$

c)  $5(3x + 4y) - 2(2x - 5y) + \frac{1}{2}(2x + 4y)$

d)  $-3(x^2 - 5x + 4) - (2x^2 - 5) + \frac{2}{3}(6x + 3)$

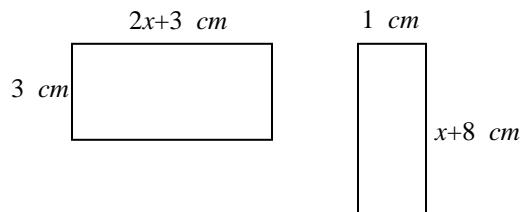
5. Simplify.

$$-0.4\left(3.\bar{3}a + 2\frac{1}{2}b\right) + \sqrt{\frac{4}{49}}\left(\frac{1}{2}a - \frac{3}{4}b\right)$$

6. Use the exponent laws to simplify the following to a single power of 2 and then evaluate if possible.

$$(2^3)^{x+5} \div (2^4)^{x+2} \div (2^{-1})^{x-1}$$

7. Given the two rectangles below, determine simplified expressions for each of the following:



a) how much more the area of the first rectangle is than the area of the second rectangle

b) how much more the perimeter of the first rectangle is than the perimeter of the second rectangle

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**Answers**

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1. a)  $15h + 10$  b)  $-24x + 18$  c)  $12x - 8y$  d)  $10a - 15b + 40c$

2. a)  $-4y - 15$  b)  $-2x - 39y$  c)  $7m^2 - 4m + 6$  d)  $-5a + 1$  e)  $-23x^2 + 52xy + 4y^2$  f)  $-15p^2q^2 + 26pq - 9$  g)  $x^2 - 2$  h)  $12x^2 - 8y^2$

i)  $-26m + 45n - 10$  j)  $2x - 11$  k)  $9x - 24$  l)  $10k + 6$  3. a)  $x + 3$  b)  $-5x^2 + 4x + 1$  c)  $-\frac{7}{2}r + \frac{5}{4}s$

4. a)  $-0.06a - 0.1b + 2$  b)  $1.1x - 1.57$  c)  $12x + 32y$  d)  $-5x^2 + 19x - 5$  5.  $-\frac{25}{21}a - \frac{17}{14}b$  6.  $2^6 = 64$  7. a)  $(5x + 1) \text{ cm}^2$  b)  $(2x - 6) \text{ cm}$

## 4.4 Multiplying a Polynomial by a Monomial (Continued)

Ex. 1. Expand and simplify.

a)  $-2y(3y + 1)$

b)  $-3n^3(3n + 1 - 2n^2)$

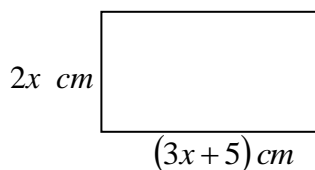
c)  $4x(3x^2 - 2x + 1) - 3x(2x^2 - 5)$

d)  $3x(4x - 5y) - 2y(2x + 3y)$

e)  $2x - 3x[5 - (2x - 1)]$

Ex. 2. a) Find a **simplified** expression for the area and perimeter of the rectangle shown.

b) Evaluate the perimeter and the area if  $x = 6 \text{ cm}$ .







## 4.4 Multiplying a Polynomial by a Monomial (Continued) Worksheet

1. Simplify.

a)  $-6x(5x)$

b)  $-5ab(-7c)$

c)  $2xy(-xy)$

d)  $2m^2(4m^2n^2)$

2. Simplify the following by expanding.

a)  $-6x(5+x)$

b)  $3x(x+y)$

c)  $-2a^3(4a^2-a)$

d)  $(3y-5)(2y)$

e)  $-b^2(2b^3-4b+1)$

f)  $-4x(x^2-3x+4)$

g)  $-2t^2(3t-5+4t^3)$

h)  $6ab^3(2a^2b-3ab^2-0.5)$

i)  $7x^2y^3(2x^2y^3+5xy^4-3x^3yz)$

3. Simplify the following by expanding and adding "like" terms.

a)  $3x^2(4xy)+3xy(-x^2)$

b)  $-2x(4x)-2x(-5)+3x(2x)+3x(-1)$

c)  $3k(5k-3)-(2k^2+3k-1)$

d)  $-2t(t+3)+4t(t-2)-5t(-2t)$

e)  $4m(m^2+4m+2)-2m(3m^2-6m+9)$

f)  $-5x(x^3-2x^2)+2x^2(3x^2-5x)-4x^3(x-2)$

g)  $-x(x+2y)+3y(x+5y)$

h)  $3a(a^2-3ab+b^2)+b(2a^2-ab)+(-2a)^3$

4. Simplify the following by expanding and adding “like” terms.

a)  $4m - 2m[m - 3(m + 2)]$       b)  $2xy(xy + 3) - x(5x + 2y - 4xy^2) - 2x[3y - (2x + 4y)]$

5. Simplify first, and then evaluate for  $x = -3$ .

a)  $-\frac{2}{3}x^2 - \frac{1}{3}x^2(4 - 3x)$

b)  $-3x(5 + x) + 2x(3x) - (3x)^2$

c)  $\frac{(3^{2x})^{x-1}}{(3^x)^{2x+3}(3^{3x})^{-2}}$       **Hint:** Use the exponent laws to simplify to a single power of base 3.

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### Answers

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1. a)  $-30x^2$    b)  $35abc$    c)  $-2x^2y^2$    d)  $8m^4n^2$    2. a)  $-6x^2 - 30x$    b)  $3x^2 + 3xy$    c)  $-8a^5 + 2a^4$    d)  $6y^2 - 10y$    e)  $-2b^5 + 4b^3 - b^2$

f)  $-4x^3 + 12x^2 - 16x$    g)  $-8t^5 - 6t^3 + 10t^2$    h)  $12a^3b^4 - 18a^2b^5 - 3ab^3$    i)  $14x^4y^6 + 35x^3y^7 - 21x^5y^4z$

3. a)  $9x^3y$    b)  $-2x^2 + 7x$    c)  $13k^2 - 12k + 1$    d)  $12t^2 - 14t$    e)  $-2m^3 + 28m^2 - 10m$    f)  $-3x^4 + 8x^3$    g)  $-x^2 + xy + 15y^2$    h)  $-5a^3 - 7a^2b + 2ab^2$

4. a)  $4m^2 + 16m$    b)  $6x^2y^2 + 6xy - x^2$    5. a)  $x^3 - 2x^2; -45$    b)  $-6x^2 - 15x; -9$    c)  $3^x; \frac{1}{27}$

### 4.5 Multiplying a Binomial by a Binomial or Trinomial

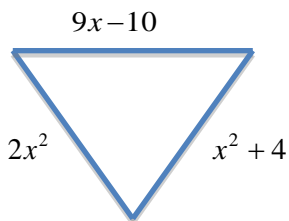
**Recall:**

Algebraic expressions can be simplified using the **Distributive Property**. The term in front of the brackets is **“shared”** with every term inside the brackets through multiplication.

**Polynomials in Geometry:      Worksheet A- Mystery Message**

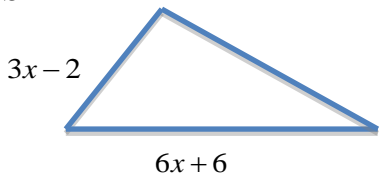
**Part 1.** Find the perimeter.

**O**



**Part 2.** Find the missing side length. The perimeter,  $P$ , is given.

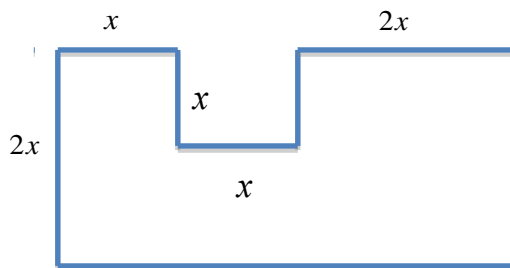
**S**



$P = 13x + 11$

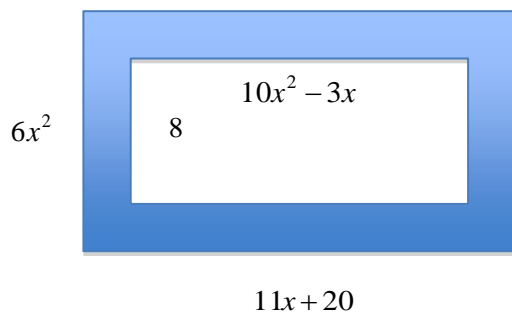
**Part 3.** Find the area.

**Q**



**Part 4.** Find the area of the shaded region.

**W**



## Multiplying a Binomial by a Binomial or Trinomial:

Using the **Distributive Property**, each term of the binomial in the first set of brackets is multiplied with each term of the binomial or trinomial in the second set of brackets.

**Ex. 1.** Simplify each of the following by expanding and then adding like terms.

a)  $(n+5)(n-6)$

b)  $(2n-3)(n-4)$

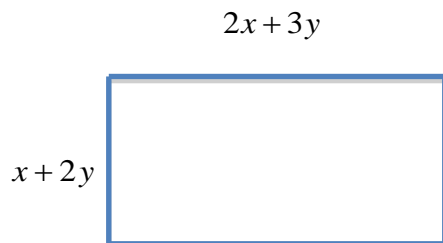
c)  $(4x-5)(6x+7)$

d)  $(3x-5y)^2$

e)  $(2t+1)(4t^2-2t+1)$

f)  $(3t+5)(t^2-3t-8)$

**Ex. 2.** Find an expression for the area of the rectangle below.

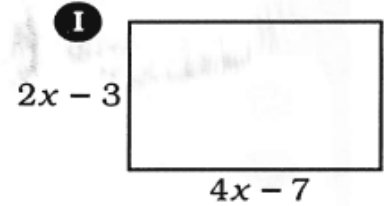
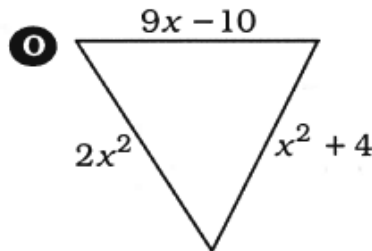
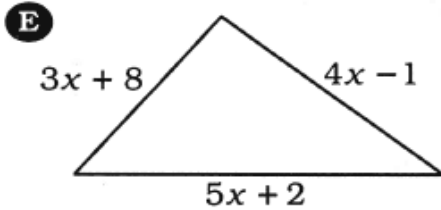


**HW:** Finish 4.5 Worksheets A and B. Show all work on lined paper by following the examples in this note.

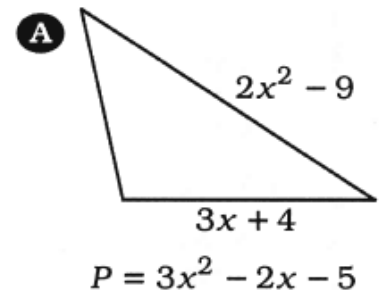
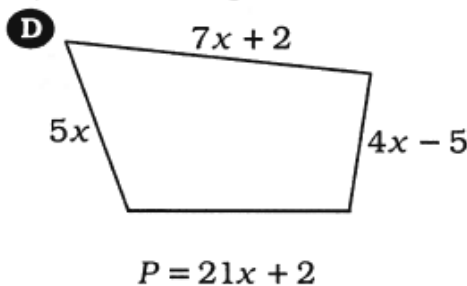
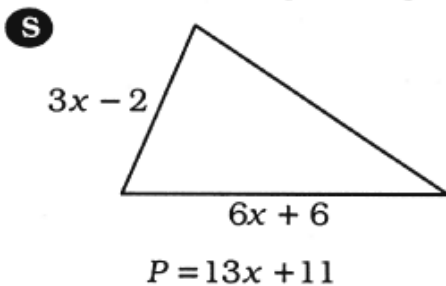
# Mystery Message

Do each exercise and find your answer at the bottom of the page. Write the letter of the exercise in the box above the answer. (Assume that figures that appear to be rectangular are rectangles.)

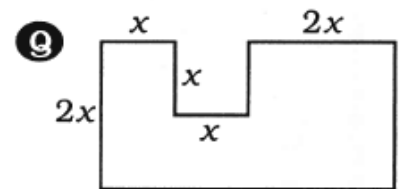
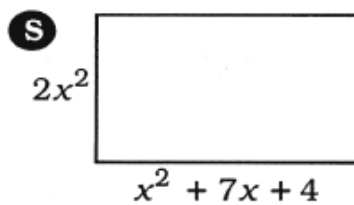
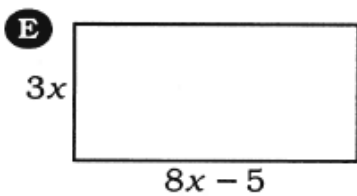
**Part 1. Find the perimeter.**



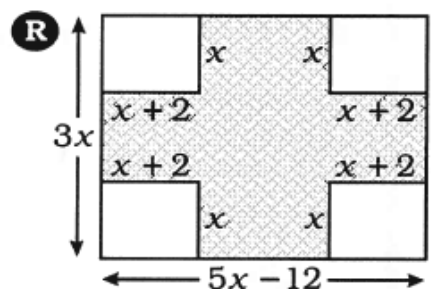
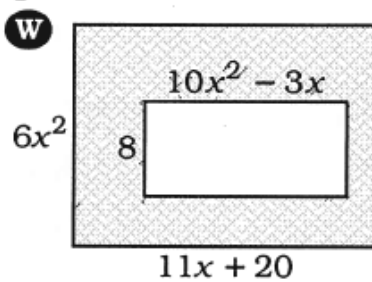
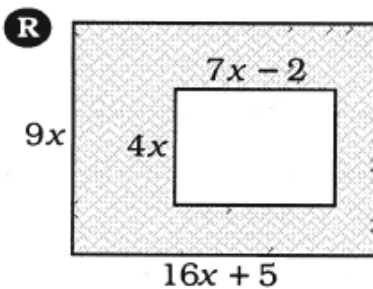
**Part 2. Find the missing side length. The perimeter, P, is given.**



**Part 3. Find the area.**



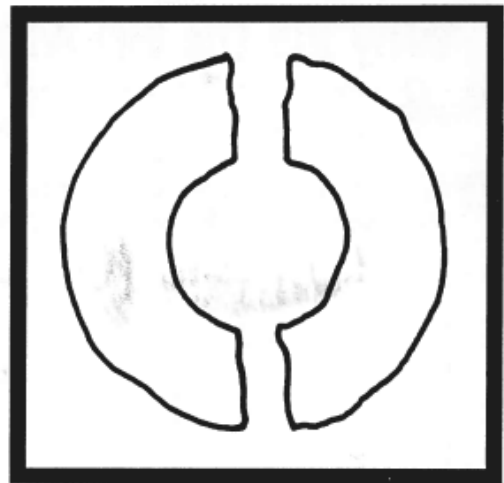
**Part 4. Find the area of the shaded region.**



$7x^2$	$4x + 7$	$9x^2 - 40x$	$x^2 - 5x$	$116x^2 + 53x$	$12x + 9$	$2x^4 + 9x^3 + 12x^2$	$66x^3 + 40x^2 + 24x$	$24x^2 - 15x$	$12x - 20$	$11x^2 - 44x$	$5x + 5$	$64x^3 + 36x^2 + 30x$	$3x^2 + 9x - 6$	$2x^4 + 14x^3 + 8x^2$	<b>??</b>
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# Law of the Donut

**What Famous Rule of Donuts Is Illustrated by This Picture?**



For the first exercise in each set, find the area of the rectangle. For all other exercises, multiply using the distributive property. Write the letter of the exercise in the box that contains the number of the answer.

**S**  $n + 5$   
 $n + 2$

**E**  $5x + 1$   
 $3x + 2$

**M**  $t^2 + 4t - 3$   
 $t + 2$

**A**  $(n + 4)(n + 9)$

**A**  $(9x - 2)(4x - 4)$

**T**  $(2t - 3)(3t^2 + 2t + 5)$

**E**  $(n - 3)(n + 10)$

**L**  $(6x + 1)(3x - 2)$

**K**  $(4t + 1)(2t^2 - 7t + 2)$

**W**  $(n - 6)(n - 5)$

**E**  $(5x - 4)(2x + 7)$

**H**  $(3t - 4)(2t^2 - t - 5)$

**O**  $(2n + 2)(3n + 8)$

**O**  $(2x + 5y)(x + 6y)$

**L**  $(8t - 3)(t^2 + 2t + 9)$

**A**  $(7n + 5)(4n - 1)$

**H**  $(4x - y)(9x - 4y)$

**V**  $(5t + 2)(4t^2 - 3t - 10)$

**Answers**

**Answers**

**Answers**

- 16  $n^2 + 7n + 36$
- 22  $n^2 + 7n - 30$
- 7  $28n^2 - 11n - 5$
- 10  $n^2 + 7n + 10$
- 2  $n^2 - 11n + 30$
- 13  $28n^2 + 13n - 5$
- 6  $n^2 + 13n + 36$
- 14  $n^2 + 22n + 30$
- 20  $6n^2 + 22n + 16$

- 3  $2x^2 + 17xy + 30y^2$
- 18  $36x^2 - 20xy + 4y^2$
- 7  $18x^2 - 9x - 2$
- 19  $36x^2 - 25xy + 4y^2$
- 4  $18x^2 + 27x - 2$
- 15  $10x^2 + 27x - 28$
- 9  $15x^2 + 13x + 2$
- 8  $10x^2 - 44x - 28$
- 17  $36x^2 - 44x + 8$

- 11  $8t^3 - 26t^2 + 60t - 27$
- 21  $8t^3 + 13t^2 + 66t - 27$
- 1  $6t^3 - 5t^2 + 4t - 15$
- 16  $20t^3 + 13t^2 - 50t - 20$
- 12  $t^3 + 6t^2 + 5t - 6$
- 8  $20t^3 - 7t^2 - 56t - 20$
- 4  $6t^3 - 5t^2 + 8t + 20$
- 14  $8t^3 - 26t^2 + t + 2$
- 5  $6t^3 - 11t^2 - 11t + 20$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
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## 4.6 Dividing a Polynomial by a Monomial and Factoring

### PART A: Dividing a Polynomial by a Monomial

**Ex. 1.** Simplify each of the following expressions.

a)  $\frac{-30x^4y^8z^2}{-6xy^2z^2}$

b)  $\frac{-4x^2}{-2} - \frac{6x}{-2} + \frac{2}{-2}$

c)  $\frac{12a^5b^2}{3a^3b^2} + \frac{9a^4b^3}{3a^3b^2} - \frac{6a^3b^4}{3a^3b^2}$

**Rule:** When dividing by a monomial, **each term** of the polynomial must be divided by the monomial.

**Ex. 2.** Divide.

a)  $\frac{-3x+9}{-3}$

b)  $\frac{5x-15y+10}{5}$

c)  $\frac{20x^2y-30xy^2+10xy}{10xy}$

d)  $\frac{24xy-16y}{8y}$

e)  $\frac{-3a^2b^2+5a^3b-4a^2b}{-a^2b}$

**Ex. 3.** Determine the **Greatest Common Factor (GCF)** of each of the following terms.

a)  $8x^2$  and  $16x$

b)  $15abc$  and  $25bc$

c)  $7x^4y^3z^2$  and  $2x^3y^2z$

## PART B: Factoring Polynomials

A **factor** is a number or term that divides evenly into each term of a polynomial. In algebra, **to factor** means to express a polynomial as a **product of its factors**, usually a **monomial**  $\times$  **polynomial**. The monomial factor is the **Greatest Common Factor, (GCF)** and the polynomial factor is the result of dividing each term in the original polynomial by the monomial factor.

**Ex. 1.** Fill in the missing information.

a)  $5x^2 + 35x$   
 $= \underline{\hspace{2cm}} (x+7)$

b)  $16x^4y^2 - 8x^2y$   
 $= 8x^2y(\underline{\hspace{2cm}})$

**Ex. 2.** Factor the following polynomials.

\*Find the GCF of all terms in the polynomial and then determine the other factor by dividing each term by the GCF.

a)  $10x - 20$

b)  $22x - 99y$

c)  $x^2 - x$

d)  $-m^2 + m$

e)  $-10x^2 - 25x$

f)  $13a + 12b - 4c$

g)  $12x^2y^2z - 8xyz$

h)  $-16x^2 + 8y + 4x$

i)  $35a^4b^3 - 21a^3b + 7a^2b^2$



# Why Did the Backpacker Carry a Flashlight?



Write the exercise letter in the box containing the number of the answer.

Find the greatest common factor.

- E**  $10x + 35$
  - O**  $4x^2 - 15x$
  - A**  $20x^2 + 36x$
  - H**  $11x^3 - 2x^2$
  - T**  $24x^3 - 64x^2 + 40x$
  - S**  $60x^4 + 70x^3 - 10x^2$
- answers
- 21**  $5x^3$
  - 28**  $4x$
  - 7**  $8x$
  - 19**  $5$
  - 3**  $2x^2$
  - 24**  $10x^2$
  - 12**  $x$
  - 17**  $x^2$

Find the greatest common factor.

- D**  $10a^2b + 16ab^2$
  - E**  $11a^3b - 2a^2b^4$
  - H**  $36a^4 + 27a^3b$
  - A**  $3a^2b^2 + 18ab^3 - 33b^4$
  - I**  $4a^4b^2 - 9a^3b^3 + a^2b^4$
  - N**  $75a^5b^3 + 30a^3b^5 - 60ab^7$
- answers
- 2**  $a^2b$
  - 5**  $3b^2$
  - 27**  $a^4b$
  - 20**  $15ab^3$
  - 22**  $9a^3$
  - 9**  $2ab$
  - 15**  $a^2b^2$
  - 4**  $5ab^2$

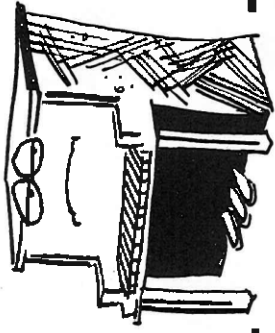
Simplify the expression.

- O**  $\frac{8t + 12}{4}$
  - T**  $\frac{2t^2 - 9t}{t}$
  - E**  $\frac{15t^2 + 20t}{5t}$
  - I**  $\frac{6t^3 - 11t^2}{t^2}$
  - H**  $\frac{4t^3 + 18t^2 - 10t}{2t}$
  - L**  $\frac{-9t^4 - 60t^3 + 3t^2}{-3t^2}$
- answers
- 23**  $6t - 11$
  - 14**  $3t^2 + 20t - 1$
  - 10**  $3t - 11$
  - 11**  $2t - 9$
  - 27**  $2t + 3$
  - 25**  $3t^2 - 10t - 1$
  - 8**  $3t + 4$
  - 1**  $2t^2 + 9t - 5$

Simplify the expression.

- W**  $\frac{8a^2b + 3ab^2}{ab}$
  - D**  $\frac{12ab^2 - 40a^2b}{4ab}$
  - G**  $\frac{7a^5b^4 + 49a^4b^5}{7a^2b}$
  - N**  $\frac{-10a^3b + 30ab^3}{-2ab}$
  - L**  $\frac{18a^7b^5 + 45a^5b^3 + 9a^3b}{9a^3b}$
  - T**  $\frac{15a^4b^4 - 3a^4b^5 - 6a^3b^6}{3a^2b^3}$
- answers
- 16**  $a^3b^3 + 7a^2b^4$
  - 18**  $5a^2b - a^2b^2 - 2ab^3$
  - 4**  $8a + 3b$
  - 21**  $2a^4b^2 + 5ab^2 + 1$
  - 6**  $5a^2 - 15b^2$
  - 26**  $2a^4b^4 + 5a^2b^2 + 1$
  - 29**  $3b - 10a$
  - 13**  $5a^2b - 2a^2b^4 + ab^3$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
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# Why Didn't the Piano Work?

Factor the expression. Find a factor in each of the two answer columns. One factor will have a letter and the other a number. Write the letter in the matching numbered box at the bottom of the page.

**Set 1**

- a.  $x^2 - 6x$
- b.  $2x^2 + 8x$
- c.  $45x^2 - 20x$
- d.  $9x^3 + 30x$
- e.  $8x^5 - 15x^3$

**Set 1 Answers**

- 11.  $x^3$
- 12.  $3x^2 + 10$
- 13.  $2x^2$
- 14.  $x - 6$
- 15.  $2x$
- 16.  $8x^2 + 15x$
- 17.  $3x$
- 18.  $9x - 4$
- 19.  $x$
- 20.  $3x^2 - 4$
- 21.  $x^2$
- 22.  $8x^2 - 15$
- 23.  $5x$
- 24.  $x + 4$

**Set 3**

- a.  $m^3n + 9m^2n$
- b.  $10m^3n^2 - 25m^2n^3$
- c.  $49m^5n^3 + 28mn^4$
- d.  $72m^7n + 24n$
- e.  $8m^3n^4 - 22m^5n^6$

**Set 3 Answers**

- 10.  $5m^2n^2$
- 11.  $4 - 11m^2n^2$
- 12.  $24n$
- 13.  $7m^4 + 4n$
- 14.  $7mn^4$
- 15.  $2m - 5mn^2$
- 16.  $m^2n$
- 17.  $3m^7 + 1$
- 18.  $2m^3n^4$
- 19.  $m + 9$
- 20.  $7mn^3$
- 21.  $4 - 11mn^3$
- 22.  $8mn$
- 23.  $2m - 5n$

**Set 2**

- a.  $8a^3 + 4a^2 + 12a$
- b.  $7a^4 - 35a^3 - 14a^2$
- c.  $6a^8 + 10a^6 - 3a^4$
- d.  $36a^3 - 24a^4 + 60a^5$
- e.  $30a^6 - 75a^5 - 15a$

**Set 2 Answers**

- 7.  $2a^2$
- 8.  $a^4$
- 9.  $15a$
- 10.  $7a^2$
- 11.  $3a^2$
- 12.  $4a$
- 13.  $2a^2$
- 14.  $3a^2$
- 15.  $2a^5 - 5a^4 - 1$
- 16.  $4a$
- 17.  $6a^4 + 10a^2 - 3$
- 18.  $12a^3$
- 19.  $3 - 2a - 5a^3$

**Set 4**

- a.  $40x^2 - 100xy - 80y^2$
- b.  $12x^5y^2 + 9x^4y^2 - 6x^3y^2$
- c.  $15x^3y - 35x^2y^2 + 40xy^3$
- d.  $144x^8y^2 + 27x^4y + 9x^2$
- e.  $2\pi x^2 - \pi y^2$

**Set 4 Answers**

- 17.  $5xy$
- 18.  $4x^2 + 3x - 6y$
- 19.  $3x^2y$
- 20.  $16x^6y^2 + 3x^2y + 1$
- 21.  $9x^2$
- 22.  $2x^2 - 5xy - 4y^2$
- 23.  $5x^2$
- 24.  $4x^2 + 3x - 2$
- 25.  $3x^3y^2$
- 26.  $2x^2 - y^2$
- 27.  $\pi$
- 28.  $3x^2 - 7xy - 4y^2$
- 29.  $20$
- 30.  $3x^2 - 7xy + 8y^2$

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## Review for Unit 4 Test

### PART A – Circle the best answer for each question.

- The total cost for a large pizza is \$14.50, plus \$1.25 for each topping. Which of the following represents the relationship between the total cost,  $C$ , in dollars, and the number of toppings,  $n$ ?  
a.  $C = 1.25n$                       b.  $C = 15.75n$                       c.  $C = 1.25n + 14.50$                       d.  $C = 14.50n + 1.25$
- A ski resort rents 300 snowboards originally priced at \$800 each. If the yearly drop in value is \$60 per snowboard, an expression for the value of the snowboards, in dollars, after  $t$  years is:  
a.  $2400 + 60t$                       b.  $300(800 - 60t)$                       c.  $2400 - 60t$                       d.  $2400 - 1800t$
- Which of the following is a simplified form of  $(-2m + 3) - (5m - 6)$ ?  
a.  $-7m - 3$                       b.  $-7m + 9$                       c.  $3m - 3$                       d.  $3m + 9$
- Which of the following is **not** of degree four?  
a.  $1 - 3x^3 + x^4$                       b.  $3xy^2 + 2xy^3 - xy$                       c.  $4x^3 + 4x^2 + 4x + 4$                       d.  $a^2b^2 + 2a$
- The simplified expression for the perimeter of a square with each side measuring  $3x - 4$  units is:  
a.  $12x - 16$  units                      b.  $12x - 4$  units                      c.  $4(3x - 4)$  units                      d.  $7x - 8$  units
- The width of a rectangle with an area of  $8x^6y^9$  cm<sup>2</sup> and a length of  $2x^3y^3$  cm is:  
a.  $16x^9y^{12}$  cm                      b.  $6x^3y^6$  cm                      c.  $4x^2y^3$  cm                      d.  $4x^3y^6$  cm
- The simplified product of  $4a^2 - 8a$  and  $-2a$  is:  
a.  $4a^2 - 10a$                       b.  $-8a^3 + 16a^2$                       c.  $-2a + 4$                       d.  $4a^2 - 6a$
- The simplified quotient of  $4a^2 - 8a$  and  $-2a$  is:  
a.  $4a^2 - 10a$                       b.  $-8a^3 + 16a^2$                       c.  $-2a + 4$                       d.  $4a^2 - 6a$
- The greatest common factor of  $32x^3y^2 - 16x^2y - 24x^3y$  is:  
a.  $2xy$                       b.  $8x^2y$                       c.  $32x^3y^2$                       d.  $8x^3y$
- Which of the following is the correct expansion for  $(3x - 5y)^2$ ?  
a.  $9x^2 - 30xy + 25y^2$                       b.  $9x^2 - 25y^2$                       c.  $6x^2 - 30xy + 8y^2$                       d.  $9x^2 + 25y^2$

**PART B – Do all questions in the space provided.**

**1. Factor.**

**a)**  $-15x - 30y$

**b)**  $12x^4 - 24x^3 + 6x^2$

**c)**  $-24mnp + 18mpq$

**d)**  $-a^2b^2 + a^2b - a^2$

**e)**  $2x^3y^4 - 6x^2y^2 + 10x^3y^5$

**f)**  $-20m^5n^2 - 15m^3n^4 + 5m^2n^3$

**g)**  $x^{12} + x^{14}$

**h)**  $2^{12} + 2^{14}$

**i)**  $y^{25} - y^{22}$

**j)**  $3^{25} - 3^{22}$

**2. Simplify.**

**a)**  $-5n(-3m^2) + 3m(-6mn)$

**b)**  $-3^2(4n) + \left(\frac{-1.5n}{0.3}\right)$

**c)**  $\frac{24x^3y}{(-2x)^3} - \frac{28y^2}{-7y}$

**d)**  $2x^2 - 5x + 5x^2 - x$

**e)**  $y^2 - (-5xy) + y^2 + (-xy)$

**f)**  $0.3x + 4.1y - 3.6 - 0.7x - 3.2y - 0.7$

**g)**  $\sqrt{\frac{4}{9}}n - 1.\bar{3}p - \frac{1}{6}n + 0.75p$

**h)**  $\frac{2}{3}\left(\frac{1}{5}x - \frac{7}{8}y + \frac{9}{4}z\right)$

**i)**  $\frac{15x^3 - 10x^2 + 20x}{5x}$

**j)**  $0.6(0.5a - 2b + 1.2c)$

**k)**  $-3x(x^2 - x)$

**l)**  $2x^2y(3xy - 4x - y)$

**3. Simplify.**

**a)**  $(3m+5)-(6m-7)+(4m-3)$

**b)**  $(3a^2-2ab+b^2)-(a^2+3ab-b^2)-(2a^2+ab+2b^2)$

**c)**  $\frac{3}{4}(16x-8y)-\frac{1}{3}(9x-3y)$

**d)**  $2x(3x-2)-(3x)^2-3x^2(x-1)$

**e)**  $6x-4x[2x-(x-3)]$

**f)**  $\frac{-30x+24y-12}{2} + \left( \frac{25x-30y-15}{5} \right)$

**g)**  $\frac{4a^3b^4-8a^2b^2}{4a^2b^2} - \frac{3a^2b^2-6a^3b^4}{3a^2b^2}$

**h)**  $-4x(3x+5y)-5(x^2+3xy)-2y(6x^2-2x)$

**i)**  $2x(5x^2-2x+7)-3x(4x^2-x-2)$

**j)**  $(x+2)(x+5)$

**k)**  $(2x+3)(3x-2)$

**l)**  $(3m+5n)(3m-8n)$

**m)**  $(2a-b)^2-(a^2+b^2)$

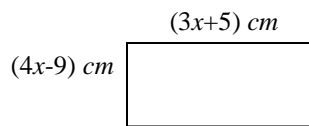
**n)**  $(3x-4y)(3x+4y)-3x^2-2y(4x-3y)$

**PART C – Do all questions on lined paper.**

1. Simplify and then evaluate for  $x = -2$ .

$$3x + 2[2x - 4x(x + 3)]$$

2. Given the rectangle below, determine:



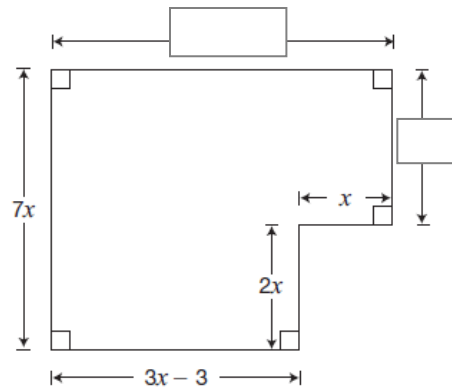
a) simplified expressions for the perimeter and area

b) the exact perimeter and area, using the simplified expressions from a) if  $x = 3$ .

3. The diagram for the floor is given and has algebraic expressions for the lengths of its sides, in metres.

Determine simplified expressions for:

- a) the lengths of the missing sides  
 b) the total perimeter of the floor,  $P$ , in  $m$ .  
 c) the area of the floor,  $A$ , in  $m^2$ .



4. A new car was purchased for \$24 400. Each year the car depreciates by \$3 100.

- a) Define the variable and use it to determine an expression that represents the remaining value of the car in the future.  
 b) Use the expression to determine what the car is worth in 5 years.

5. Simplify to a single power using the exponent laws, and then evaluate.

a)  $4^{2x} \div 4^{x-5} \div 4^{x+8}$

b)  $\left(3^{\frac{1}{2}}\right)^{4x-2} \times \left(3^{\frac{1}{4}}\right)^{4-8x}$

c)  $8^x \div 2^{3x-4}$

6. Evaluate the following by first factoring the numerator and denominator and then reducing.

\*HINT: refer to PART B #1 h), j) for factoring help.

a)  $\frac{5^{20} - 5^{18}}{5^{18} + 5^{19}}$

b)  $\frac{2^{99} + 2^{102}}{2^{101} - 2^{103}}$

**Answers**

**PART A**

1. c 2. b 3. b 4. c 5. a 6. d 7. b 8. c 9. b 10. a

**PART B**

1. a)  $-15(x + 2y)$  b)  $6x^2(2x^2 - 4x + 1)$  c)  $-6mp(4n - 3q)$  d)  $-a^2(b^2 - b + 1)$  e)  $2x^2y^2(xy^2 - 3 + 5xy^3)$  f)  $-5m^2n^2(4m^3 + 3mn^2 - n)$   
 g)  $x^{12}(1 + x^2)$  h)  $2^{12}(1 + 2^2)$  or  $2^{12}(5)$  i)  $y^{22}(y^3 - 1)$  j)  $3^{22}(3^3 - 1)$  or  $3^{22}(26)$

2. a)  $-3m^2n$  b)  $-4ln$  c)  $y$  d)  $7x^2 - 6x$  e)  $2y^2 + 4xy$  f)  $-0.4x + 0.9y - 4.3$  g)  $\frac{1}{2}n - \frac{7}{12}p$  h)  $\frac{2}{15}x - \frac{7}{12}y + \frac{3}{2}z$  i)  $3x^2 - 2x + 4$   
 j)  $0.3a - 1.2b + 0.72c$  k)  $-3x^3 + 3x^2$  l)  $6x^3y^2 - 8x^3y - 2x^2y^2$

3. a)  $m + 9$  b)  $-7ab$  c)  $9x - 5y$  d)  $-3x^3 - 4x$  e)  $-4x^2 - 6x$  f)  $-10x + 6y - 9$  g)  $3ab^2 - 3$  h)  $-12x^2y - 17x^2 - 31xy$   
 i)  $-2x^3 - x^2 + 20x$  j)  $x^2 + 7x + 10$  k)  $6x^2 + 5x - 6$  l)  $9m^2 - 9mn - 40n^2$  m)  $3a^2 - 4ab$  n)  $6x^2 - 8xy + 22y^2$

**PART C**

1.  $-8x^2 - 17x$ ; 2 a)  $P = 14x - 8$  cm;  $A = 12x^2 - 7x - 45$  cm<sup>2</sup> b)  $P = 44$  cm;  $A = 42$  cm<sup>2</sup>

3. a)  $4x - 3$ ;  $5x$  b)  $P = 22x - 6$  m c)  $A = 26x^2 - 21x$  m<sup>2</sup>

4. a) Let  $t$  represent the number of years since the car was new. Value of car in \$:  $24400 - 3100t$  b) \$8900

5. a)  $4^{-3} = \frac{1}{64}$  b)  $3^0 = 1$  c)  $2^4 = 16$  6. a) 4 b)  $-\frac{3}{4}$