5.1 Solving Simple Equations

Warm-up:

Solve the following by inspection.

1. $x+5=13$

$x=8$

2. $-3x=-4x-5$

?

3. $0.8n+18.3=3.9$

?

Solving Equations by: "Isolating the Variable through Inverse Operations"

Ex. 1. Solve each equation using inverse operations.

a) $x+5=13$

$x+5-5=13-5$

$x=8$

b) $-3x=-4x-5$

$-3x+4x=-4x-5+4x$

$x=-5$

Shortcut: "Transposing terms using inverse operations"

Variable terms go on the left side of the equation and constants go on the right side.

If you switch sides, you switch operations.

Ex. 2. Solve each equation by transposing terms through inverse operations first.

a) $x+5=13$

$x=13-5$

$x=8$

b) $-3x=-4x-5$

$-3x+4x=-5$

$x=-5$

c) $10+2y=30$

$2y=30-10$

$\frac{2y}{2}=\frac{20}{2}$ $\longrightarrow$ $y=\frac{20}{2}$

$y=10$ $\quad y=10$

d) $2y-8=3y$

$2y-3y=8$

$\frac{-y}{-1}=\frac{8}{-1}$

$y=-8$

e) $3x-1=14$

$3x=14+1$

$\frac{3x}{3}=\frac{15}{3}$

$x=5$

f) $5y-6=-6$

$5y=-6+6$

$\frac{5y}{5}=\frac{0}{5}$

$y=0$

g) $5-5x=x-15$

$-5x-x=-15-5$

$\frac{-6x}{-6}=\frac{-20}{-6}$

$x=\frac{10}{3}$

h) $2y=8+2y$

$2y-2y=8$

$\frac{0y}{0}=\frac{8}{0}$ undefined

or $0=8$

$\therefore$ there is no solution

i) $1.2n+18.3=0.4n+3.9$

$1.2n-0.4n=3.9-18.3$

$\frac{0.8n}{0.8}=\frac{-14.4}{0.8}$ $\times 10$ $\times 10$

$n=\frac{-144}{8}$

$n=-18$

$18.3-3.9$

$8\overline{)144}$

Ex. 3. **Solve** and do a **formal check** using a "**L.S. = R.S.=** format" for the following equation:

$-6x+7=-15-4x$

$-6x+4x=-15-7$

$\frac{-2x}{-2}=\frac{-22}{-2}$

$x=11$

Check $x=11$

$LS=-6x+7$

$=-6(11)+7$

$=-66+7$

$=-59$ ✓

$RS=-15-4x$

$=-15-4(11)$

$=-15-44$

$=-59$ ✓

$\therefore LS=RS$

$\therefore x=11$ is the solution.

Ex. 4. For the following word problems write an equation that models the situation, and then solve. Remember to define the variable with a "Let" statement first.

a) Colin ordered 3 pizzas. If he paid $27.00 for the order including a delivery cost $1.50, how much was each pizza?

Let x represent the cost of one pizza, in $.

$3x+1.50=27.00$

or

$3x+1.5=27$

$3x=27-1.5$

$\frac{3x}{3}=\frac{25.5}{3}$

$x=8.5$

$\therefore$ Each pizza cost $8.50.

$$3\overline{)25.5}\quad 8.5$$
$$-24$$
$$15$$
$$-15$$
$$0$$

b) A number decreased by one is equal to three more than seven times the number. Determine the number.

Let x represent the unknown number.

$x-1=7x+3$

$x-7x=3+1$

$\frac{-6x}{-6}=\frac{4}{-6}$

$x=-\frac{2}{3}$

$\therefore$ the number is $-\frac{2}{3}$.

HW: Worksheet 5.1

5.2 Solving Multi-Step Equations

Ex. 1. Solve and formally check the following equation.

Remember to transpose terms so that variable terms are on one side of the equation and constants are on the other.

$5x-6=2x+8$

$5x-2x=8+6$

$\frac{3x}{3}=\frac{14}{3}$

$\therefore x=\frac{14}{3}$

(no decimals; leave in fraction form)

Check

$LS=5x-6$

$=\frac{5}{1}(\frac{14}{3})-\frac{6}{1}$

$=\frac{70}{3}-\frac{18}{3}$

$=\frac{52}{3}$

$RS=2x+8$

$=\frac{2}{1}(\frac{14}{3})+\frac{8}{1}$

$=\frac{28}{3}+\frac{24}{3}$

$=\frac{52}{3}$

$\because LS=RS$

$\therefore x=\frac{14}{3}$ is the solution.

Ex. 2. Solve each equation.

Make sure each side is simplified before transposing terms.

a) $\frac{-24+6x}{-3}=3x-17$

$\frac{-24}{-3}+\frac{6x}{-3}=3x-17$

$8-2x=3x-17$

$-2x-3x=-17-8$

$\frac{-5x}{-5}=\frac{-25}{-5}$

$x=5$

b) $2x-5-3x+1=7x-8-2x$

$-x-4=5x-8$

$-x-5x=-8+4$

$\frac{-6x}{-6}=\frac{-4}{-6}$

$x=\frac{2}{3}$

c) $14-3(5t-12)=1-(20t+1)$

$14-15t+36=1-20t-1$

$-15t+50=-20t$

$-15t+20t=-50$

$\frac{5t}{5}=\frac{-50}{5}$

$t=-10$

d) $-\frac{1}{2}(6x-14)-8x=4-2(x+3)$

$-3x+7-8x=4-2x-6$

$-11x+7=-2x-2$

$-11x+2x=-2-7$

$\frac{-9x}{-9}=\frac{-9}{-9}$

$x=1$

e) $(2k-5)(3k+1)=3k(2k-1)$

$6k^2+2k-15k-5=6k^2-3k$

$6k^2-13k-5=6k^2-3k$

$6k^2-6k^2-13k+3k=5$ ($6k^2-6k^2=0$)

$\frac{-10K}{-10}=\frac{5}{-10}$

$K=-\frac{1}{2}$

Ex. 3. Use an **equation** to determine the value of x and **state any angle properties** used in your solution.

$(3x+10)+2(x+5)=180$

$3x+10+2x+10=180$

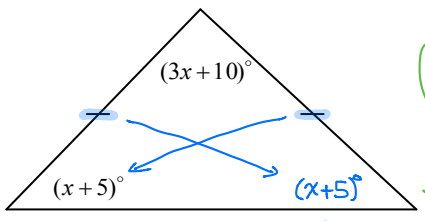
$5x+20=180$

$5x=180-20$

$\frac{5x}{5}=\frac{160}{5}$

$x=32$

$5\overline{)160}$ → 32, -15, 10, -10, 0



all interior angles in a △ add to 180°

isosceles △ has 2 equal angles across from the equal sides.

HW: 5.2 Worksheets A and B

Date: April 2/19

Solving Multi-Step Equations Continued

Ex. 1. Solve and formally check the following equation.

$-(2x+1) = -7 - 7(x+2)$

$-2x-1 = -7-7x-14$

$-2x-1 = -7x-21$

$-2x+7x = -21+1$

$\frac{5x}{5} = \frac{-20}{5}$

$\therefore x = -4$

* must follow BEDMAS order

Check $x=-4$

$LS = -(2x+1)$
$= -[2(-4)+1]$
$= -(-8+1)$
$= -1(-7)$
$= 7$

$RS = -7-7(x+2)$
$= -7-7[(-4)+2]$
$= -7-7(-2)$
$= -7+14$
$= 7$

$\because LS = RS$
$\therefore x = -4$ is the solution

Ex. 2. Write an equation in terms of one variable to model each situation below and solve.

a) Liam sells sandwiches at an arena. He earns $10.50 per hour plus $0.40 for each sandwich he sells How many sandwiches does he need to sell during a 6-hour shift to earn $125?

Let x *represent how many sandwiches Liam needs to sell during a 6-hour shift to earn $125.*

$0.40x + 10.50(6) = 125$

$0.4x + 63 = 125$

$0.4x = 125-63$

$\frac{0.4x}{0.4} = \frac{62}{0.4}$ (×10 on both)

$x = \frac{620}{4}$

$x = 155$

$4\overline{)620}$: -4↓, 22, -20↓, 20, -20, 0 → 155

$\therefore$ Liam needs to sell 155 sandwiches to earn $125.

b) A submarine is currently submerged at a depth of 600 m below sea level. If it rises at a rate of 4 m/s, how long will it take for the submarine to reach a depth of 486 m below sea level?

Let x *represent how long it will take to reach a depth of 486 m below sea level, in seconds.*

$-600 + 4x = -486$

$4x = -486+600$

$\frac{4x}{4} = \frac{114}{4}$

$x = 28\frac{2}{4}$

$x = 28\frac{1}{2}$

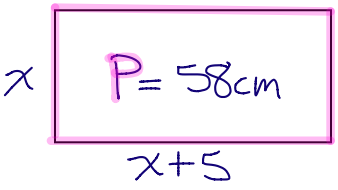
$600 - 486 = 114$

$4\overline{)114}$: -8↓, 34, -32, 2 → 28

$\therefore$ It will take $28\frac{1}{2}$ seconds to reach 486m below sea level.

c) The perimeter of a rectangle is 58 cm. If the length is 5 cm longer than the width, find the rectangle's dimensions.

Let x *represent the width of the rectangle in* cm.
Let $x+5$ *represent the length of the rectangle in* cm.



$P = 2w + 2l$
$58 = 2(x) + 2(x+5)$
$58 = 2x + 2x + 10$
$58 = 4x + 10$
$-4x = 10 - 58$
$\frac{-4x}{-4} = \frac{-48}{-4}$
$x = 12$

∴ the rectangle's width is 12 cm and length is 17 cm.

d) The Sun Spa charges annual dues of $125 plus $10 per hour to use the facilities. The Moon Spa charges annual dues of $230 plus $7 per hour to use the facilities. For what number of hours would the two spas charge the same total amount?

Let x *represent the number of hours for which the two spas charge the same total amount.*

$10x + 125 = 7x + 230$
$10x - 7x = 230 - 125$
$\frac{3x}{3} = \frac{105}{3}$
$x = 35$

$3\overline{)105}$ → 35, $-9\downarrow$, 15, -15, 0

∴ both spas charge the same amount for 35 hours.

e) Simon says: "Five times my age 4 years ago is the same as 3 times my age in 2 years." How old is Simon now?

Let x *represent Simon's age now in years.*

$5 \times Simon's\ age_{4\ yrs\ ago} = 3 \times Simon's\ age_{in\ 2\ yrs}$
$5(x-4) = 3(x+2)$
$5x - 20 = 3x + 6$
$5x - 3x = 6 + 20$
$\frac{2x}{2} = \frac{26}{2}$
$x = 13$

∴ Simon is currently 13 years old.

HW: 5.3 Worksheet

 Date: April 3/19

5.4 Solving Equations Involving Fractions

Rule:

Clear all fractions by multiplying **all terms** on **both sides** of the equation by the **lowest common denominator**.

Ex. 1. Solve each equation.

a) $\frac{x}{4} = -10$

$\frac{1}{1}\left(\frac{x}{4}\right) = \frac{4}{1}\left(\frac{-10}{1}\right)$

$x = -40$

b) $\frac{x}{10} = \frac{3}{5}$

$\frac{10}{1}\left(\frac{x}{10}\right) = \frac{2}{1}\left(\frac{3}{5}\right)$

$x = 6$

c) $\frac{2x}{3} + 1 = -4$

$\frac{1}{3}\left(\frac{2x}{3}\right) + \frac{3}{1}\left(\frac{1}{1}\right) = -\frac{3}{1}\left(\frac{4}{1}\right)$

$2x + 3 = -12$

$2x = -12 - 3$

$\frac{2x}{2} = -\frac{15}{2}$

$x = -\frac{15}{2}$

d) $\frac{4y}{3} - \frac{1}{2} = 4$

$\frac{2}{6}\left(\frac{4y}{3}\right) - \frac{3}{6}\left(\frac{1}{2}\right) = \frac{6}{1}\left(\frac{4}{1}\right)$

$8y - 3 = 24$

$8y = 24 + 3$

$\frac{8y}{8} = \frac{27}{8}$

$y = \frac{27}{8}$

Ex. 2. Solve and check.

$\frac{2n}{3} - \frac{1}{2}n = 7 + \frac{3}{4}n$

$\frac{4}{12}\left(\frac{2n}{3}\right) - \frac{6}{12}\left(\frac{1}{2}n\right) = \frac{12}{1}\left(\frac{7}{1}\right) + \frac{3}{12}\left(\frac{3}{4}n\right)$

$8n - 6n = 84 + 9n$

$2n = 84 + 9n$

$2n - 9n = 84$

$\frac{-7n}{-7} = \frac{84}{-7}$

$n = -12$

Check $n = -12$

$LS = \frac{2n}{3} - \frac{1}{2}n$

$= \frac{2(-12)}{3} - \frac{1}{2}(-12)$

$= -\frac{24}{3} + \frac{12}{2}$

$= -8 + 6$

$= -2$

$RS = 7 + \frac{3}{4}n$

$= 7 + \frac{3}{4}(-12)$

$= 7 - \frac{36}{4}$

$= 7 - 9$

$= -2$

$\therefore LS = RS$

$\therefore n = -12$ is correct.

HW: Worksheet 5.4

Date: April 4/19

5.5 Solving Equations Involving Fractions Continued

Recall:
Clear all fractions by multiplying **everything** on **both sides** of the equation by the **lowest common denominator**.

Ex. 1. Solve each equation.

a) $\frac{3x}{4} - 7 = 2\frac{1}{2}x$

$\frac{3x}{4} - 7 = \frac{5}{2}x$

$\frac{1}{4}\left(\frac{3x}{4}\right) - \frac{4}{1}\left(\frac{7}{1}\right) = \frac{4}{1}\left(\frac{5}{2}x\right)$

$3x - 28 = 10x$

$3x - 10x = 28$

$\frac{-7x}{-7} = \frac{28}{-7}$

$x = -4$

b) $\frac{y-5}{4} = \frac{2y+7}{3}$

$\frac{3}{12}\left(\frac{y-5}{4}\right) = \frac{4}{12}\left(\frac{2y+7}{3}\right)$

$3(y-5) = 4(2y+7)$

$3y - 15 = 8y + 28$

$3y - 8y = 28 + 15$

$\frac{-5y}{-5} = \frac{43}{-5}$ $\quad y = -\frac{43}{5}$

c) $\frac{x+3}{3} - \frac{2-x}{7} = 1$

$\frac{7}{21}\left(\frac{x+3}{3}\right) - \frac{3}{21}\left(\frac{2-x}{7}\right) = \frac{21}{1}\left(\frac{1}{1}\right)$

$7(x+3) - 3(2-x) = 21$

$7x + 21 - 6 + 3x = 21$

$10x + 15 = 21$

$10x = 21 - 15$

$\frac{10x}{10} = \frac{6}{10}$

$x = \frac{3}{5}$

d) $\frac{2}{3}(x+2) - \frac{5}{6}(x-2) = 3$

$\frac{2}{6}\left[\frac{2}{3}(x+2)\right] - \frac{1}{6}\left[\frac{5}{6}(x-2)\right] = \frac{6}{1}\left(\frac{3}{1}\right)$

$4(x+2) - 5(x-2) = 18$

$4x + 8 - 5x + 10 = 18$

$-x + 18 = 18$

$-x = 18 - 18$

$\frac{-x}{-1} = \frac{0}{-1}$

$x = 0$

Ex. 2. Solve and check.

$\frac{m+3}{2} - 4 = \frac{4-m}{3} + 7$ ← 4 terms

$\frac{3}{6}\left(\frac{m+3}{2}\right) - 6(4) = \frac{2}{6}\left(\frac{4-m}{3}\right) + 6(7)$

$3(m+3) - 24 = 2(4-m) + 42$

$3m + 9 - 24 = 8 - 2m + 42$

$3m - 15 = -2m + 50$

$3m + 2m = 50 + 15$

$\frac{5m}{5} = \frac{65}{5}$

$m = 13$

Check

$LS = \frac{m+3}{2} - 4$

$= \frac{(13)+3}{2} - 4$

$= \frac{16}{2} - 4$

$= 8 - 4$

$= 4$

$RS = \frac{4-m}{3} + 7$

$= \frac{4-(3)}{3} + 7$

$= -\frac{9}{3} + 7$

$= -3 + 7$

$= 4$

$\therefore LS = RS$

$\therefore m = 13$ is the solution.

Ex. 3. Given the formula to convert Fahrenheit temperatures to Celsius is $C = \frac{5}{9}(F - 32)$, convert:

a) $5^\circ F$ to $^\circ C$

Find C if F=5

$C = \frac{5}{9}[F-32]$

$C = \frac{5}{9}[(5)-32]$ evaluate

$C = \frac{5}{9_1}\left(\frac{-27^{-3}}{1}\right)$

$C = -15$

∴ 5°F is −15°C

b) $20^\circ C$ to $^\circ F$

Find F if C=20

$C = \frac{5}{9}(F-32)$

$20 = \frac{5}{9}(F-32)$ solve

$\frac{9}{1}\left(\frac{20}{1}\right) = \frac{9}{1}\left[\frac{5}{9_1}(F-32)\right]$

$180 = 5(F-32)$

$180 = 5F - 160$

$-5F = -160 - 180$

$\frac{-5F}{-5} = \frac{-340}{-5}$ $5\overline{)340}$

$F = 68$

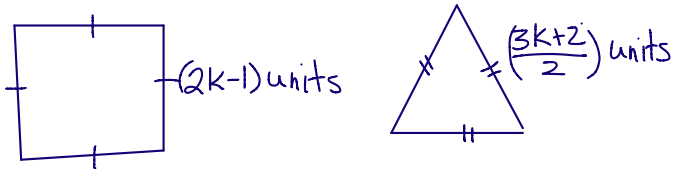
∴ 20°C is 68°F

Ex. 4. A square has sides of length $2k-1$ units. An equilateral triangle has sides of length $\frac{3k+2}{2}$ units.

The square and the triangle have the same perimeter.

a) Find the value of k.

b) Find the perimeter.



$P_{square} = P_{triangle}$

a) $4(2k-1) = 3\left(\frac{3k+2}{2}\right)$

$2[4(2k-1)] = \frac{2}{1}\left[3\left(\frac{3k+2}{2_1}\right)\right]$

$8(2k-1) = 3(3k+2)$

$16k - 8 = 9k + 6$

$16k - 9k = 6 + 8$

$\frac{7k}{7} = \frac{14}{7}$

$K = 2$

b) Find P if k=2

$P = 4(2k-1)$

$P = 4[2(2)-1]$

$P = 4(4-1)$

$P = 4(3)$

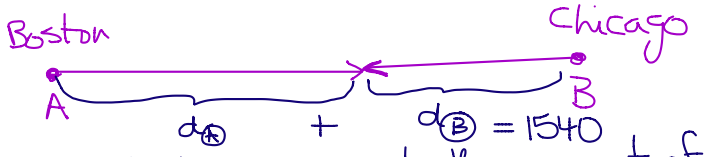
∴ P = 12 units.

HW: 5.5 Worksheets A and B

 Date: April 5/19

5.6 Word Problems

Ex. 1. One train leaves Boston for Chicago travelling at 80 km/h and a second train leaves Chicago for Boston travelling at 60 km/h. The distance from Boston to Chicago is 1540 km.
a) How many hours after they left each station will the two trains meet?
b) How far will the train bound for Boston have travelled?



distance = speed x time

a) Let x represent the amount of time, in hours.

$80x + 60x = 1540$

$\frac{140x}{140} = \frac{1540}{140}$

$x = \frac{154}{14}$

$x = 11$

$14\overline{)154}$ → -14 → 14 → -14 → 0 ; quotient 11

∴ they meet after 11 hours of travel.

b) If $x = 11$: $d_B = 60(11)$
$= 660$

∴ the train bound for Boston traveled 660 km.

Ex. 2. One half of a certain even integer plus one fifth of the next consecutive even integer equals 48. Find the two integers.

Let x represent an even integer, and x+2 is the next consecutive even integer.

$\frac{1}{2}x + \frac{1}{5}(x+2) = 48$ or $\frac{x}{2} + \frac{x+2}{5} = 48$

$\frac{5}{10}\left(\frac{1}{2}x\right) + \frac{2}{10}\left[\frac{1}{5}(x+2)\right] = 10(48)$ | $\frac{5}{10}\left(\frac{x}{2}\right) + \frac{2}{10}\left(\frac{x+2}{5}\right) = 10(48)$

$5x + 2(x+2) = 480$

$5x + 2x + 4 = 480$

$7x = 480 - 4$

$\frac{7x}{7} = \frac{476}{7}$

$x = 68$

If $x = 68$ then $x + 2 = 70$.

∴ the two even integers are 68 & 70.

HW: 5.6 Worksheets A and B to be completed on lined paper.
Good solutions are required with "let" statements and "concluding" statements.